How Strong is the Weak Axiom?

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Introduction

- For sufficiently rich choice/demand data sets, possible inconsistencies with preference maximization are characterized by simple pairwise, or local consistency conditions.

- This paper is concerned with what constitutes a ‘rich enough’ collection of observations for such results, *independent of the specific choices observed.*

Motivation

- *Experimental*: What experiments preclude the existence of testable implications of preference transitivity beyond pairwise-coherent choices?
- *Applied Theory*:
  - *Behavioral*: How far can full-domain assumptions in choice models be relaxed?
  - *Measurement of Irrationality*: How does structure of choice problem create potential dependencies between choice cycles?
- *Computational*: When does it suffice to verify the absence of choice cycles by checking only for 2-cycles?

Abstract Choice I

A **choice problem** is a pair (X, Σ), where:

- X is a set of **alternatives**.
- $\Sigma \subseteq 2^X \setminus \{\varnothing\}$ is a collection of non-empty subsets of X called **budgets**.

These budgets correspond to the subsets of X from which we observe the agent choose.

- Assumptions on Σ are assumptions on *observability*.

Abstract Choice II

A **choice correspondence** is a map $c : \Sigma \to 2^X \setminus \{\varnothing\}$ satisfying:

$$(\forall B \in \Sigma) \quad c(B) \subseteq B.$$

The **revealed preference pair** associated to c, denoted $(\succsim_c, \succ_c)$, is defined via:

- $x \succsim_c y$ if there exists a budget $B \in \Sigma$ such that $\{x, y\} \in B$, and $x \in c(B)$.
- $x \succ_c y$ if there exists a budget $B \in \Sigma$ such that $\{x, y\} \in B$, $x \in c(B)$, and $y \notin c(B)$.

Rationalizable Choice

A choice correspondence c is **strongly rationalizable** if there exists a weak order $\succeq$ on X such that:

$$(\forall B \in \Sigma) \quad c(B) = \left\{x \in B : \forall y \in B,\ x \succeq y\right\}$$

The -ARPs

- A choice correspondence satisfies the **weak** axiom of revealed preference (WARP) if it makes no choice *reversals*:

$$x \succsim_c y \implies y \not\succ_c x.$$

- It satisfies the **generalized** axiom of revealed preference (GARP) if it contains no finite choice *cycles* of the form:

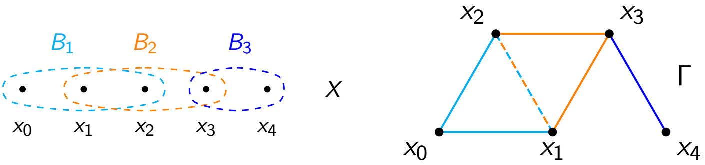
$$x_0 \succsim_c x_1 \succsim_c \cdots \succsim_c x_{N-1} \succ_c x_0.$$

The Budget Graph

For a choice problem (X, Σ) its **budget graph** Γ is the undirected graph with vertex set $V_\Gamma = X$ and edge set:

$$E_\Gamma = \left\{ \{x, y\} \subseteq X : \exists B \in \Sigma \text{ s.t. } \{x, y\} \subseteq B \right\}.$$

An Example



(a) A choice problem. **(b)** The budget graph.

Figure: A choice problem with five alternatives and three budgets.

Cyclic Collections

For a loop $\gamma = (V_\gamma, E_\gamma)$, a collection of budgets $\mathcal{B}_\gamma \subseteq \Sigma$ is a **cyclic collection** for γ if:

$$(\forall e \in E_\gamma)\,(\exists B \in \mathcal{B}_\gamma) \quad e \subseteq B.$$

A cyclic collection $\mathcal{B}_\gamma$ for a loop γ is **covered** if there exists a budget $\bar{B} \subseteq \cup_{\tilde{B} \in \mathcal{B}_\gamma} \tilde{B}$ that either:

(i) Contains V_γ; or

(ii) Contains a pair of elements of V_γ that are not connected by an edge in E_γ.

Note: Condition (i) implies (ii) if and only if $|V_\gamma| > 3$.

Well-covered Budget Collections

A budget collection Σ is **well-covered** if, for every loop γ in its budget graph, every cyclic collection for γ is covered.

Theorem

Let (X, Σ) be a choice problem. The weak axiom of revealed preference is characteristic of strong rationalizability if and only if Σ is well-covered.

Discussion

Well-coveredness characterizes when choice cycles imply choice reversals.

It is a *richness* condition, rather than a measure of data set size:

- Adding budgets to a well-covered budget collection may create uncovered cyclic collections.
- Removing budgets may remove uncovered cyclic collections.

Intro to Integrability

The problem of integrability of a demand function is parallel to that of the strong rationalizability of a choice correspondence.

Hurwicz, Uzawa et al. tell us a (nice) demand arises from constrained-optimal choice according to a (nice) utility if and only if its Slutsky matrix is:

- Negative semi-definite $\iff$ weak axiom; and
- Symmetric $\iff$ locally integrable.

Complete domain: we know x for every (p, w).

Domains

Let (X, Σ) a be a choice problem with budget graph $\Gamma = (X, E_\Gamma)$.

- The **domain** associated to the choice problem is the triple (X, E_Γ, T_Γ) where:

$$T_\Gamma = \big\{\{x, y, z\} \subseteq X : \{x, y\}, \{y, z\}, \{x, z\} \in E_\Gamma\big\}.$$

- Given some collection $\tilde{T} \subseteq T_\Gamma$, the **subdomain** generated by $\tilde{T}$ is the collection of 1-, 2-, and 3-element subsets of elements of $\tilde{T}$.

Local Rationalizability

A choice correspondence is **locally rationalizable** if there exists an order extension $(\succeq, \succ)$ of $(\succsim_c, \succ_c)$ such that:

$$(\forall \tau \in T_\Gamma) \quad \succeq|_\tau \text{ is complete and transitive.}$$

Simple (Sub)domains

A (sub)domain $(\tilde{X}, \tilde{E}, \tilde{T})$ is **simple** if it is:

(i) *Combinatorially Trivial*: If $\tau, \tau' \in \tilde{T}$, there is a unique, finite sequence of distinct elements of $\tilde{T}$:

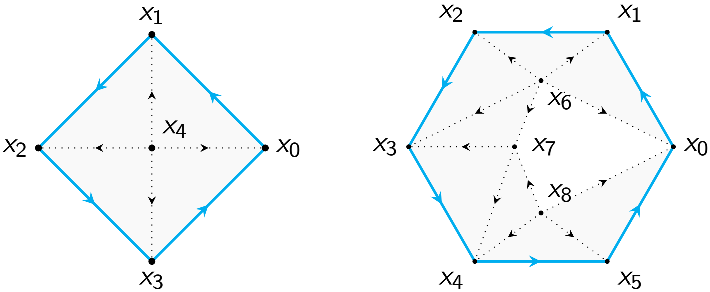
$$\tau = \tau_0, \tau_1, \ldots, \tau_k = \tau'$$

such that τ_j and τ_{j+1} share precisely a pair of elements.

(ii) *Topologically Trivial*: The (sub)domain has a first Betti number of zero.

By abuse of notation, we say a *domain* is simple if every loop in Γ is contained in a simple subdomain.

The Triviality Conditions



(a) A locally rational binary relation on a topologically trivial domain that is combinatorially non-trivial.

(b) A locally rational binary relation on a combinatorially trivial domain that is topologically non-trivial.

Figure: On non-simple (sub)domains, locally rational binary relations may not be acyclic.

Ordinal Integrability

Theorem

Let (X, Σ) be a choice problem with a simple domain. Then a choice correspondence is strongly rationalizable if and only if:

(i) *It obeys the weak axiom; and*

(ii) *It is locally rationalizable.*

Moreover, (i) and (ii) are equivalent to strong rationalizability if and only if the domain of (X, Σ) is simple.

Discussion

In comparison with the classical Hurwicz-Uzawa theory of integrability:

- Still require the weak axiom and a local 'no cycles' condition.
- Complete domain assumption not needed, only simplicity.
- No analytic, point-set, or order theoretic assumption on primitives required.

Integrability Within the Empirical Content of Choice

Tradeoff: in classical theory, extra assumptions yield nice structure on rationalizing preference. Here only a weak order.

- The price for a theory that imposes no assumptions that are non-falsifiable by finite data sets.

A Decomposition of Well-coveredness

Integrability theory provides an *economic* interpretation of well-coveredness.

- *Local*: Any revealed preference pair obeying the weak axiom is locally rationalizable.
  - "Negative semi-definite Slutsky matrix $\implies$ symmetric."
- *Global*: The domain of the choice problem is simple, hence these local conditions characterize strong rationalizability.
  - The set of observations is sampled "richly" enough.

Conclusions

- The objective of this paper is to weaken the informational requirements of choice theory.
- A great deal of economic theory, both classical and modern, supposes an exhaustive collection of observations.
- This is a first step in studying just how widely applicable 'full domain' theories are in practice, and how they may be adapted to hold as widely as possible.

Thank you!

Any Questions?