The Implications of Experimental Design for Choice Data

Pete Caradonna

Georgetown University

ppc14@georgetown.edu

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Introduction

- Cycles in revealed preference data are often thought of as fundamental units of choice-theoretic inconsistency.
- However, choice cycles are generally not independent of each other.
- The particular collection of budgets we observe choice on has strong implications for structure of potential inconsistency: cyclic choices over certain alternatives often force cyclic choices over others.

An Example

- Suppose $X = \{x_1, x_2, x_3, x_4\}$. We observe choice on budgets $\{x_1, x_2\}$, $\{x_2, x_3\}$, $\{x_3, x_4\}$, $\{x_4, x_1\}$ and $\{x_1, x_2, x_3\}$.

- Consider choices:
  - $c(\{x_1, x_2\}) = \{x_1\}$
  - $c(\{x_2, x_3\}) = \{x_2\}$
  - $c(\{x_3, x_4\}) = \{x_3\}$
  - $c(\{x_4, x_1\}) = \{x_4\}$.

- What about choice on $\{x_1, x_2, x_3\}$?

An Example

Suppose we work with singleton-valued choice correspondences...

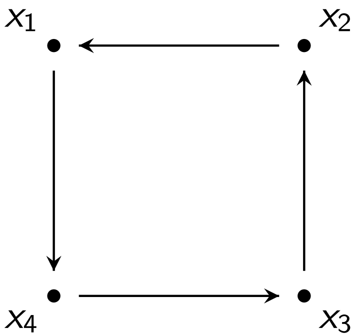

Figure: The only choice from $\{x_1, x_2, x_3\}$ that doesn't create a reversal relative to the choices on $\{x_1, x_2\}$ and $\{x_2, x_3\}$ is to choose $\{x_1\}$. But this choice creates another cycle too: $x_1 \succ_c x_3 \succ_c x_4 \succ_c x_1$.

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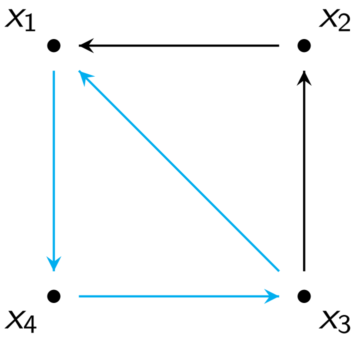

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Takeaway

- **Every** choice over $\{x_1, x_2, x_3\}$ must create another cycle.
- In fact, **every** choice correspondence that chooses

$$x_1 \succ_c x_2 \succ_c x_3 \succ_c x_4 \succ_c x_1$$

  chooses in at least one other additional cycle. We say that the potential cycle has the **propagation property**.

A Tradeoff

- **Fundamental Tension**: The more exhaustive the set of budgets we observe choice on, the richer our understanding of an agent's behavior, but the harder it becomes to interpret and measure inconsistency.

Related Results

- Propagation-free environments exist, but are degenerate in a way we make precise. Most experiments will suffer from possibility of some propagation.
- Propagation holding 'uniformly' is necessary and sufficient for the weak axiom of revealed preference to characterize rationalizability (in the sense of Richter).

Application: Structure of Inconsistency

- In all but very sparse experiments, choice cycles can propagate.
- This means not all choice cycles should necessarily be treated independently. Some may be ‘explainable’ by others.
- Measures of irrationality *should account for the structure of the environment*.

A Relation On Cycles

- Suppose we conduct a (finite) experiment where we observe a choice correspondence c.
- Let $\mathscr{Z}$ denote the collection of all observed revealed preference cycles.
- Say z **explains** z' if choices making up z' either (i) also make up part of z or (ii) are on budgets such that, given the choices making up z, any choice would yield another cycle.

An Example: Revisited

Consider our choice function from earlier, with $c(\{x_1, x_2, x_3\}) = \{x_1\}$.

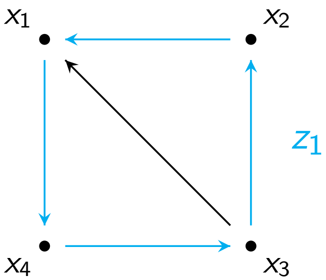

Figure: All choices making up z_2 either (i) also make up z_1 or (ii) are on budgets such that given the choices making up z_1 any choice yields another cycle, thus z_1 explains z_2.

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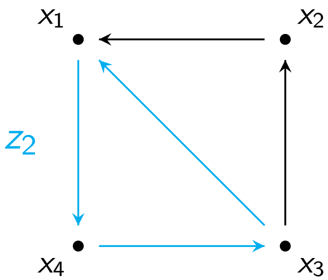


Figure: All choices making up z_2 either (i) also make up z_1 or (ii) are on budgets such that given the choices making up z_1 any choice yields another cycle, thus z_1 explains z_2.

The Irrational Kernel

A collection of cycles $\mathcal{I} \subseteq \mathcal{Z}$ is an **irrational kernel** of the data if:

(i) *Explanatory Power*: Every cycle in $\mathcal{Z}$ is (at least indirectly) explained by some cycle in $\mathcal{I}$.

(ii) *Independence*: No two cycles in $\mathcal{I}$ (even indirectly) explain each other.

The ‘Rank’ of $\mathcal{Z}$

- Irrational kernels may not be unique. But every irrational kernel has the same cardinality.
  - **Intuition**: Irrational kernel is like ‘maximal independent set’ of cycles, given the structure of the choice environment.
- Size of irrational kernel is a principled refinement of simply counting cycles: how many *independent* cycles explain the inconsistency.

Data

We apply this theory to the data set of Harbaugh, Krause, and Berry (2001 AER).[1]

A Few Observations:

- For irrational subjects, vast majority have some ‘dependent’ cycles.
- For subjects with ‘lots’ of cycles, irrational kernel is generally far smaller (order of magnitude).
- Observe reversals relative to naïvely counting cycles when comparing relative degree of rationality between agents.

[1]We thank Bill Harbaugh for generously sharing this data.

Conclusions

- The structure of a choice experiment can have strong implications for the interpretation of potential inconsistency. Specifics of which budgets we observe choice on matters!
- We characterize how the structure of an experiment may cause cycles to propagate, and how to account for this phenomenon in data.

Thank you!

Any Questions?

Technical Appendix

Choice Environments

A **choice environment** is a pair (X, Σ), where:

- X is a set of **alternatives**.
- $\Sigma \subseteq 2^X \setminus \{\varnothing\}$ is a collection of non-empty subsets of X called **budgets**.

These budgets correspond to the subsets of X from which we observe the agent choose.

- Assumptions on Σ are assumptions on *observability*.

Choice Data

A **choice correspondence** is a map $c : \Sigma \to 2^X \setminus \{\varnothing\}$ satisfying:

$$(\forall B \in \Sigma) \quad c(B) \subseteq B.$$

The **data set** associated with a choice correspondence c is:

$$\left\{B, c(B)\right\}_{B \in \Sigma}.$$

In particular, we assume we observe the budget each choice arises from.

Revealed Preferences

The **revealed preference pair** associated to c, denoted $(\succsim_c, \succ_c)$, is defined via:

- $x \succsim_c y$ if there exists a budget $B \in \Sigma$ such that $x, y \in B$, and $x \in c(B)$.
- $x \succ_c y$ if there exists a budget $B \in \Sigma$ such that $x, y \in B$, $x \in c(B)$, and $y \notin c(B)$.

The -ARPs

- A choice correspondence satisfies the **weak** axiom of revealed preference (WARP) if it makes no choice *reversals*:

$$x \succsim_c y \implies x \not\succ_c y.$$

- It satisfies the **generalized** axiom of revealed preference (GARP) if it contains no finite choice *cycles* of the form:

$$x_0 \succsim_c x_1 \succsim_c \cdots \succsim_c x_{N-1} \succ_c x_0.$$

Rationalizable Choice

- A choice correspondence c is **strongly rationalizable** if there exists a weak order $\succeq$ on X such that:

$$(\forall B \in \Sigma) \quad c(B) = \left\{x \in B : \forall y \in B,\ x \succeq y\right\}$$

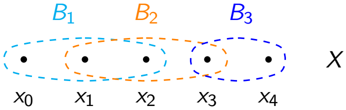
- *(Richter '66 Ecta)*: A choice correspondence is strongly rationalizable if and only if it satisfies GARP.

The Budget Graph

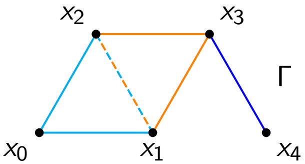
For a choice problem (X, Σ) its **budget graph** Γ is the undirected graph with vertex set $V_\Gamma = X$ and edge set:

$$E_\Gamma = \left\{ \{x, y\} \subseteq X : \exists B \in \Sigma \text{ s.t. } \{x, y\} \subseteq B \right\}.$$

An Example



(a) A choice environment.



(b) The budget graph.

Figure: A choice environment with five alternatives and three budgets.

Cyclic Collections

For a loop $\gamma = (V_\gamma, E_\gamma)$, a collection of budgets $\mathcal{B}_\gamma \subseteq \Sigma$ is a **cyclic collection** for γ if:

(i) Every edge in γ is contained in some budget in B_γ,

$$(\forall e \in E_\gamma)\,(\exists B \in \mathcal{B}_\gamma) \quad e \subseteq B.$$

(ii) Every budget in $\mathcal{B}_\gamma$ contains at least one edge of γ,

$$(\forall B \in \mathcal{B}_\gamma)\,(\exists e \in E_\gamma) \quad e \subseteq B.$$

Coverage

A cyclic collection $\mathcal{B}_\gamma$ for a loop γ is **covered** if there exists a budget $\bar{B} \subseteq \cup_{\tilde{B} \in \mathcal{B}_\gamma} \tilde{B}$ that either:

(i) Contains V_γ; or

(ii) Contains a pair of elements of V_γ that are not connected by an edge in E_γ.

Note: Condition (i) implies (ii) if and only if $|V_\gamma| > 3$.

Propagation of Choice Cycles

A loop γ has the **propagation property** if every choice correspondence that chooses cyclically around γ necessarily makes another choice cycle elsewhere in the data.

- Ex-ante property of an experiment
- Confounds interpretation of inconsistency

A Characterization of Propagation

Theorem

A loop in the budget graph has the propagation property if and only if all of its cyclic collections are covered.

Experimental Design

Propagation makes interpretation of inconsistency difficult. Can we remedy this with careful design of experiments?

Theorem

Let (X, Σ) be a choice environment, with $|X| < +\infty$. Suppose that no loop in the budget graph capable of supporting a choice cycle has the propagation property. Then every loop γ in the budget graph has a unique cyclic collection satisfying exactly one of the following:

(i) $\mathcal{B}_\gamma$ *consists of a single budget; or*

(ii) $\mathcal{B}_\gamma$ *consists exclusively of two-element budgets.*

The Power of the Weak Axiom

- GARP is necessary and sufficient for strong rationalizability, no matter the structure of (X, Σ).
- The power of WARP relative to GARP varies drastically with the structure of (X, Σ). WARP becomes 'stronger' when Σ is 'richer.'
- Only handful of examples of what constitutes a 'rich' choice environment. Poor understanding of what constitutes 'richness' for sampling.

A General Richness Condition

Call a budget collection Σ **well-covered** if, for every loop γ in its budget graph, every cyclic collection for γ is covered.

- Well-coveredness means propagation occurs uniformly: every loop in the budget graph has the propagation property.
- Recursive flavor: a covering budget for one loop implies there is a bisecting edge in the budget graph. Resulting sub-loops must also have all their cyclic collections covered.

Well-covered Budget Collections

Well-coveredness is the weakest experimental richness condition that makes WARP and GARP coincide.

Theorem

Let (X, Σ) be a choice environment. The weak axiom of revealed preference is characteristic of strong rationalizability if and only if Σ is well-covered.

Generators for Cycles

Let $\mathcal{Z}$ denote the set of all choice cycles in a given data set. A collection of budgets $\mathcal{G}_z \subseteq \Sigma$ is a **generator** for a cycle z if:

(i) For every relation $x_i \succsim_c x_{i+1}$ (resp. $x_i \succ_c x_{i+1}$) in the cycle, there exists a $B \in \mathcal{G}_z$ such that $x_i, x_{i+1} \in B$ and $x_i \in c(B)$ (resp. $x_i \in c(B)$ and $x_{i+1} \notin c(B)$).

(ii) For every $B \in \mathcal{G}_z$, there is some $x_i, x_{i+1} \in B$ with $x_i \in c(B)$, and if $x_i \succ_c x_{i+1}$ then additionally $x_{i+1} \notin c(B)$.

A Dependence Relation

- For two choice cycles $z, z' \in \mathcal{Z}$, we say z **directly explains** z', denoted $z \Longrightarrow z'$, if there exist generators for the cycles $\mathcal{G}_z, \mathcal{G}_{z'} \subseteq \Sigma$ such that:

$$\mathcal{G}_{z'} \subseteq \mathcal{G}_z \cup \left\{ B \in \Sigma : B \text{ covers } \mathcal{G}_z \right\}.$$

- Given only those choices made on budgets in $\mathcal{G}_z$, every choice on a covering budget necessarily forces another cycle.

The Irrational Kernel

Consider the transitive closure of $\Longrightarrow$ on $\mathcal{Z}$, denoted $\Longrightarrow^*$. We call a collection $\mathcal{I} \subseteq \mathcal{Z}$ an **irrational kernel** for the data if:

(i) *Explanatory Power*: For all $z' \in \mathcal{Z}$ there exists a $z \in \mathcal{I}$ such that:

$$z \Longrightarrow^* z'.$$

(ii) *Independence*: For all distinct $z, z' \in \mathcal{I}$, $z \not\Longrightarrow^* z'$.

The 'Rank' of $\mathcal{Z}$

For a particular data set, there may not be a unique irrational kernel. However, the *size* of the irrational kernel always well-defined.

Theorem

Let (X, Σ) be a choice experiment with $|X| < +\infty$. Then for any choice correspondence, there exists an irrational kernel. Moreover, every irrational kernel has the same cardinality.