Mergers, Entry, and Consumer Welfare[*]

Peter Caradonna[†]
California Institute of Technology

Nathan H. Miller[‡]
Georgetown University

Gloria Sheu[§]
Federal Reserve Board

February 14, 2024

Abstract

We model merger-induced entry in the context of differentiated-products price competition. We fully characterize the combinations of merger efficiencies and entrant qualities that can mitigate the adverse equilibrium welfare effects of an otherwise anticompetitive merger. The possibility of merger-induced entry introduces non-monotonicity into the equilibrium value that consumers receive from merger efficiencies, potentially necessitating the joint analysis of efficiencies and entry in merger review. We also explicitly characterize the efficiencies required for merger-induced entrants to make profitable mergers consumer welfare-neutral. We provide empirical applications to the US beer market and the T-Mobile/Sprint merger.

JEL Codes: K21, L13, L41

Keywords: mergers, entry, repositioning, merger efficiencies, antitrust policy

---

1   Introduction

The effect that a merger between competitors has on consumers depends on the entry decisions of other firms (e.g., Stigler, 1950). Accordingly, in merger review, antitrust authorities consider whether entry might mitigate adverse effects that arise from a loss of competition.[1] For practical purposes, this inquiry has been empirical. Antitrust authorities collect information about entry barriers and whether prospective entrants have the resources and capabilities needed to compete. They also examine whether entry has been observed previously in response to changes in demand and supply conditions.

In this paper, we reexamine the theoretical underpinnings of this exercise and provide formal frameworks for the merger review of entry that, relative to the traditional approach, are more closely connected to economic theory and offer sharper insights. Our primary results are twofold. First, we characterize the magnitude of entry and efficiencies (e.g., cost reductions) necessary to offset the adverse effects of an otherwise anticompetitive merger. Indeed, we show that entry and efficiencies are sufficiently intertwined that, for many mergers, the current practice of analyzing each in isolation is inappropriate. Second, we characterize the parametric regimes under which (i) a merger occurs without inducing entry (i) a merger induces entry but nonetheless reduces consumer surplus, and (iii) a merger yields pro-competitive outcomes because it induces entry. Our results pertain to subgame perfect equilibria and hence are consistent with backward-inductive reasoning by all firms.

We contribute to a broader shift in merger review toward modeling that uses economic theory to interpret empirical evidence (e.g., Shapiro, 2010; Miller and Sheu, 2021). Our formal analysis places a useful discipline on the traditional approach to the merger review of entry. For example, we identify instances in which merger-induced entry *cannot* lead to pro-competitive outcomes in the absence of efficiencies. The frameworks we introduce also allow for an assessment of tradeoffs when countervailing forces are at work. A leading example is a merger that eliminates a competitor but also improves operational efficiency and creates scope for profitable entry. Another arises when a loss of competition (from the merger) and merger efficiencies have opposing effects on the profitability of entry.

More concretely, in Section 2 we consider a model of merger-induced entry that features differentiated-products Bertrand competition and multinomial logit (MNL) or constant elasticity of substitution (CES) demand. We explicitly allow for the possibility that the merger generates efficiencies for the firms involved, such as marginal cost reductions or product qual-

---

[1]In the European Union and the United States, this analysis follows formal merger guidelines promulgated by their respective antitrust agencies. Both the EU and US guidelines propose the "timely, likely, and sufficient" standard for determining whether entry may serve to counteract anticompetitive actions on the part of the merging firms. For the EU, see Section 6 of the Guidelines on the Assessment of Horizontal Mergers under the Council Regulation on the Control of Concentrations between Undertakings. For the US, see §3.2 of the 2023 Merger Guidelines of the Department of Justice (DOJ) and Federal Trade Commission (FTC).

ity improvements.[2] These efficiencies increase the profitability of the merger, raise consumer surplus, and reduce the profitability of entry.[3] We provide a complete characterization of the merger, entry, and welfare outcomes generated by the (unique, subgame perfect) equilibrium of our model. We are helped in this endeavor by the aggregative games structure of the Bertrand MNL and CES models, as developed in Nocke and Schutz (2018).

We prove that, in this context, a profitable merger occurring with certain combinations of efficiencies and entry *can* increase consumer surplus, even if neither entry nor efficiencies would offset the adverse competitive effects of the merger on its own. Although this possibility depends jointly upon (i) the competitive structure of the pre-merger market, (ii) the magnitude of the efficiencies, and (iii) the competitive strength of the potential entrant, there exists a minimum threshold level of efficiencies that must be attained if the merger is to remain profitable despite entry. An implication is that if such efficiencies are unlikely to materialize, then it may be appropriate to infer that entry barriers would deter entry, as otherwise the merger would not be profitable. We show that the minimum threshold for efficiencies can be fully characterized with pre-merger market shares.

We also establish that consumer welfare is *non-monotone* in the level of the efficiencies generated by the merger. Intuitively, there exist 'just-so' levels of efficiencies that are large enough to deter post-merger entry but that are too small to offset the adverse competitive effects of the merger. Again, these effects can be evaluated using pre-merger market shares. The interplay between entry and efficiencies suggests that a joint analysis may be appropriate for many mergers, and the model provides a framework for such an analysis.

In Section 3, we extend our results to the nested analogues of the MNL and CES demand systems (the NMNL and NCES, respectively), and to random coefficients logit (RCL) demand. In the nested models, we prove that if entry is to preserve consumer surplus after a profitable merger that does not generate efficiencies, then the entrant must be a distant competitor of the merging firms.[4] Here, consumers benefit from entry mainly due to the increased product variety, rather than its effect on greater price competition. A natural tension emerges in the context: the more distant the prospective entrant would be as competitor, the smaller is the effect of the merger on the profitability of entry. Thus, in a regime where merger-induced entry *could* mitigate consumer harm without rendering the merger unprofitable, the scope for profitable entry is reduced because entry costs would need to fall into a narrower range.

The RCL demand system can incorporate rich heterogeneity in consumer preferences and

---

[2]See §3.3 of the Merger Guidelines. The analysis of efficiencies is a standard part of merger review, and numerous results are available in the literature for specific contexts (e.g. Ashenfelter et al., 2015; Braguinsky et al., 2015; Kulick, 2017; An and Zhao, 2019; Dermirer and Karaduman, 2022).

[3]That efficiencies reduce the scope for profitable entry has been explored previously in different modeling contexts (e.g, Cabral (2003); Erkal and Piccinin (2010)).

[4]More formally, the entrant must be in a different nest than the merging firms, and the parameter that damps consumer substitution between nests must be sufficiently large.

is frequently used in the empirical literature (e.g., Berry et al., 1995; Nevo, 2001), but it does not generate an aggregative game structure. Thus, we turn to numerical methods. We obtain results consistent with our earlier analyses: absent efficiencies, if a merger is to be profitable despite entry that prevents consumer surplus loss, the entrant must be sufficiently differentiated from the merging firms. To demonstrate the value of our framework in merger review, we consider the MillerCoors merger in the US beer industry. We use the demand system of Miller and Weinberg (2017) and assume Bertrand competition.[5] We show that, for technologically plausible specifications of entrant marginal costs and qualities, no entrant would restore consumer surplus without making the merger unprofitable.

We also extend our analysis to explore in greater depth the likelihood with which mergers induce entry. Because the prospective entrant in our model makes decisions that are consistent with backward-inductive reasoning, it takes into account that its entry would intensify price competition.[6] For merger-induced entry to occur, entry costs must be large enough to deter entry without the merger, but small enough that entry following the merger is profitable. This suggests model-implied bounds on entry costs that could inform how likely entry is to occur after a merger. Tighter bounds obtain if the products of the entrant would be distant substitutes for those of the merging firms. We present simulation evidence that, absent efficiencies and with MNL demand, mergers create roughly a 6%-7% profit increase for the compensating entrant per 500 point increase in the Herfindahl-Hirschman Index (HHI).[7]

Finally, in Section 4, we apply our framework to the T-Mobile/Sprint merger. For background, a US Federal District Court ruled that the merger between the mobile wireless operators could proceed, in part due to the expectation that DISH would enter the market. We calibrate a Bertrand MNL model using publicly-available data and a market elasticity of demand that appears in regulatory filings, and assume that DISH would replicate the product offerings of Sprint. Counterfactual simulations indicate that there is no equilibrium with both the merger and merger-induced entry. The merger is unprofitable if it causes DISH to enter, unless merger efficiencies are large, in which case the merger does not cause DISH to enter.

---

[5]Although MillerCoors was structured as a joint venture, it was functionally equivalent to a merger in the US, and it has been analyzed as such in several articles (e.g., Ashenfelter et al., 2015; Miller and Weinberg, 2017). Our analysis does not incorporate coordinated effects or merger efficiencies.

[6]This reduces the scope for entry relative to a model of naivety in which entrants make decision based on post-merger prices that do no not reflect the impact of entry. The most recent iteration of the Merger Guidelines (see §3.2) recognizes this line of thinking explicitly:

> "Firms make entry decisions based on the market conditions they expect once they participate in the market. If the new entry is sufficient to counteract the merger's effect on competition, the Agencies analyze why the merger would induce entry that was not planned in pre-merger competitive conditions."

[7]The compensating entrant has just enough competitive significance to exactly restore pre-merger consumer surplus (Section 2). The concept is analogous to the compensating efficiency that has been the focus of previous research (e.g., Werden, 1996; Nocke and Whinston, 2022) and antitrust practice.

Thus, the model indicates that merger-induced entry by DISH does not occur, and points to efficiencies being the more important consideration.

We conclude in Section 5 with a summary and a discussion of directions for research.

1.1 Literature Review

Our research builds on a handful of articles that consider the relationship between mergers and entry. At a high level, our incremental contribution relative to this literature derives from (1) the observation that entry and efficiencies may best be analyzed jointly, (2) our analysis of multiple demand systems, and (3) the frameworks for empirical analysis that flow from our theoretical results. These aspects of our contribution, considered together, indicate that entry analysis can connected usefully to the overall investigation of competitive effects.

Werden and Froeb (1998) examines Bertrand competition with MNL and NMNL demand. Using numerical simulations, it finds that most mergers are unprofitable if entry occurs, and furthermore that mergers do not increase the entrant's profit by much. Our results are consistent with those of Werden and Froeb, but they are sharper, more general, and (mostly) are proven analytically using the aggregative games framework of Nocke and Schutz (2018).[8] Also along these lines, Spector (2003) examines mergers and entry in a Cournot model with the general assumptions of Farrell and Shapiro (1990), and proves that mergers without efficiencies are unprofitable if merger-induced entry restores consumer surplus.

Anderson et al. (2020) examines mergers under the free entry assumption that fringe firms endogenously participate in the market both pre- and post-merger.[9] In a class of aggregative games that nests Cournot competition and our model of Bertrand competition with MNL demand, Anderson et al. establishes that a merger without efficiencies reduces the long run profit of the merging firms. As entry is taken as a given, and entry exactly restores pre-merger consumer surplus, this is essentially the flip-side of our result that mergers without efficiencies are unprofitable if they induce entry sufficient to restore pre-merger consumer surplus. Our primary contributions relative to Anderson et al lie in our consideration of richer demand systems, our analysis of efficiencies, and the empirical frameworks that we provide.

We also note that a number of articles have examined merger-induced entry using structural methods to estimate entry costs (Li et al. (2022); Ciliberto et al. (2021); Fan and Yang (2023)). Post-merger equilibrium then can be computed allowing for entry. Similarly, Starc and Wollman (2023) examines entry in response to collusion. We view such studies as complementary to our theoretical research. First, these articles employ the RCL model of Berry

---

[8]With NMNL demand, Werden and Froeb focus on a single value of the nesting parameter and assume that entry occurs in the nest of the merging firms. Our results indicate that these are meaningful restrictions. We also examine CES, NCES, and RCL demand.

[9]See also Davidson and Mukherjee (2007).

et al. (1995), so many of our results should extend. Second, our theoretical approach informs the magnitude of entry costs (or fixed costs) that could generate merger-induced entry, whereas the empirical approach informs the realized magnitude of those costs.

2 A Model of Mergers and Entry

2.1 Model Overview and Timing

We consider a three-stage game of perfect information. There are firms $f = 1, 2, \ldots, F$, with $F \geq 3$. Without loss of generality, the first $F - 1$ firms are incumbents, and final firm F is a prospective entrant. The timing of the game is as follows:

1. Firms 1 and 2 decide whether to merge to form the combined firm, M. A merger commits these firms to maximize joint profits when setting prices in Stage 3. It might also create efficiencies gains for the merging firms, which we formalize later.

2. Firm F observes whether the merger occurs in stage 1 and decides whether to enter. If it enters, it incurs a fixed entry cost, $\chi > 0$, the value of which is commonly known.

3. All firms observe whether the merger and entry occur in stages 1 and 2. The incumbents and, if entry occurs, the entrant, form the set $\mathcal{F}$. The firms in $\mathcal{F}$ choose prices simultaneously, consumers make purchasing decisions, and firms earn variable profit according to differentiated-products Bertrand equilibrium.

In adopting this three-stage structure, we follow the theoretical literature on mergers and entry (e.g., Werden and Froeb, 1998; Spector, 2003). In Appendix B, we consider an alternative structure with delayed or probabilistic entry, following the logic of Stigler (1950), and obtain numerical results that are similar to the analytical results presented in this section.

Our solution concept is subgame perfect equilibrium (SPE). Whether merger and entry occur in equilibrium is determined by the payoffs available to firms in the pricing stage of the game. The interesting case for antitrust enforcement is that of *merger-induced entry*, which we define as entry that occurs if and only if the merger occurs. This requires $\Pi_{nm}^{F} < \chi \leq \Pi_{m}^{F}$, where Π^{F} is the profit of the entrant and the subscripts nm and m refer to "no merger" and "merger," respectively. Indeed, if the entry decision is unaffected by the merger decision, then in merger review the increase in competition due to entry is not typically balanced against the loss of competition due to the merger.

An important consequence of subgame perfect reasoning is that the possibility that entry can have a deterrence effect on mergers. If the merging firms know that entry would occur if and only if they merge, then they may be dissuaded from doing so. This leads to a key revealed preference implication: if a merger is observed in practice, the merging parties must

believe barriers prohibit the entry of sufficiently competitive outsiders. In some relevant models of price competition (i.e., Stage 3), this inference about the *profitability* of a merger permits inferences on the merger's *welfare* implications: if merger-induced entry eliminates the adverse effects of the merger on consumer welfare, then it necessarily also renders the merger unprofitable unless efficiencies are sufficiently large. Our first main results provide a complete characterization of when such inferences may be drawn. While our approach is theoretical, the conditions that we will obtain are deterministic functions of pre-merger observables, and hence applicable in practice.

2.1.1 Bertrand Equilibrium in the Pricing Subgame

Throughout, we focus on assumptions that are commonly maintained in the industrial organization literature and employed in antitrust practice. On the supply-side, we assume that firms compete in prices and that marginal costs are constant. The profit functions take the form

$$\Pi^f(p) = \sum_{j \in \mathcal{J}^f} (p_j - c_j) q_j(p), \tag{1}$$

where $\mathcal{J}^f$ is the set of products sold by firm f, p_j and c_j are the price and marginal cost of product j, and $q_j(p)$ represents the quantity demanded of product j as a function of all prices. The first order condition for profit maximization for any $j \in \mathcal{J}^f$ is

$$q_j(p) + \sum_{k \in \mathcal{J}^f} (p_k - c_k) \frac{\partial q_k(p)}{\partial p_j} = 0. \tag{2}$$

Prices that satisfy this equation for all products constitute a Bertrand equilibrium of the pricing subgame.

In this section, we focus on multinomial logit (MNL) and constant elasticity of substitution (CES) demands.[10] Apart from their common use in antitrust practice, these demand systems also provide a useful theoretical baseline because they exhibit the independence of irrelevant alternatives (IIA) property. As a consequence, entry does not affect the relative market shares of incumbents. Relatedly, the diversion ratio from any product k to any other product j,

$$\frac{\frac{\partial s_j}{\partial p_k}}{\frac{\partial s_k}{\partial p_k}} \equiv DIV_{k \to j} = \frac{s_j}{1 - s_k},$$

is proportional to product j's market share.[11] We call products "close substitutes" if $DIV_{k \to j} >$

---

[10]In Section 3.1 and Section 3.2 we extend these results to nested demand systems including NMNL, NCES and RCL demands.

[11]See Miller and Sheu (2021) for a discussion of how diversion ratios are employed in merger review and

$\frac{s_j}{1-s_k}$ and "distant substitutes" if $DIV_{k \to j} < \frac{s_j}{1-s_k}$. With this terminology, the entrant and the merging firms are neither close nor distant competitors with MNL and CES. We later consider demand systems that allow for more flexible substitution patterns (Section 3).

With MNL demand, each consumer purchases a single product j from the set $\mathcal{J}$ or forgoes a purchase by selecting the outside good ($j = 0$). The indirect utility that consumer i receives from product $j \in \mathcal{J}$ is

$$u_{ij} = v_j - \alpha p_j + \epsilon_{ij}, \tag{3}$$

where v_j and p_j are the quality and price of product j, α is a price coefficient, and ϵ_{ij} is a consumer-specific preference shock. The indirect utility provided by the outside good is $u_{i0} = \epsilon_{i0}$, where we apply the standard normalization $v_0 = p_0 = 0$. The preference shocks are iid with a Type I extreme value distribution, which yields a closed-form solution for market shares (in terms of unit sales):

$$s_j(p) = \frac{\exp(v_j - \alpha p_j)}{1 + \sum_{k \in \mathcal{J}} \exp(v_k - \alpha p_k)}. \tag{4}$$

Mapping back to the profit function of equation (1), we have $q_j(p) = s_j(p)M$, where M is the density of consumers. We normalize M to one for simplicity.

With CES demand, consumer utility takes the form

$$u_i = \left(\sum_{j \in \mathcal{J}, j=0} v_j^{\frac{1}{\sigma}} q_j^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where q_j is the quantity consumed of product j, v_j is that product's quality, and $\sigma > 1$ is the elasticity of substitution between products in the utility function. Consumers choose quantities to maximize utility subject to a budget constraint of $\sum_j p_j q_j = Y$, where Y is total income. We apply a standard set of normalizations: $v_0 = p_0 = 1, Y = 1$. This obtains a closed form solution for market shares (in terms of revenue):

$$s_j(p) = \frac{v_j p_j^{1-\sigma}}{1 + \sum_{k \in \mathcal{J}} v_k p_k^{1-\sigma}}. \tag{5}$$

Mapping back to the profit function, we have $q_j(p) = s_j(p)/p_j$.

We represent Bertrand equilibrium using the *type aggregation* property of these demand systems (Nocke and Schutz (2018)).[12] In particular, equilibrium outcomes depend on a firm-level primitive—the firm type—that summarizes the qualities and marginal costs of each

Conlon and Mortimer (2021) for a useful theoretical analysis.

[12]See also Nevo and Rossi (2008), which proposes that the type aggregation property of Bertrand/logit models can help facilitate the estimation and simulation of dynamic games.

firm's products. The types take the form:

$$T^f \equiv \begin{cases} \sum_{j \in \mathcal{J}^f} \exp(v_j - \alpha c_j) & \text{(MNL)} \\ \sum_{j \in \mathcal{J}^f} v_j c_j^{1-\sigma} & \text{(CES)}. \end{cases} \tag{6}$$

Further, with these demand systems, each firm finds it optimal to apply the same markup to all of its products. It is convenient to define firm-specific "ι-markups":

$$\mu^f \equiv \begin{cases} \alpha(p_j - c_j) & \forall j \in \mathcal{J}^f \quad \text{(MNL)} \\ \sigma \frac{p_j - c_j}{p_j} & \forall j \in \mathcal{J}^f \quad \text{(CES)}. \end{cases} \tag{7}$$

By inspection, the ι-markups are proportional to the actual markups, either in levels for MNL or percentages for CES.

The Bertrand equilibrium can be characterized as a vector of ι-markups, $\{\mu^f\} \; \forall f \in \mathcal{F}$, a vector of firm-level market shares, $\{s^f\} \; \forall f \in \mathcal{F}$, and a market aggregator, H. A firm's market share is the combined market share of its products, $s^f = \sum_{j \in \mathcal{J}^f} s_j$, and the aggregator is the denominator from equations (4) and (5),

$$H \equiv \begin{cases} 1 + \sum_{j \in \mathcal{J}} \exp(v_j - \alpha p_j) & \text{(MNL)} \\ 1 + \sum_{j \in \mathcal{J}} v_j p_j^{1-\sigma} & \text{(CES)}. \end{cases} \tag{8}$$

In equilibrium, the ι-markups satisfy

$$1 = \begin{cases} \mu_f \left(1 - \frac{T^f}{H} \exp(-\mu_f)\right) & \text{(MNL)} \\ \mu^f \left(1 - \frac{\sigma-1}{\sigma} \frac{T^f}{H} \left(1 - \frac{\mu^f}{\sigma}\right)^{\sigma-1}\right) & \text{(CES)}. \end{cases} \tag{9}$$

Let the unique solution for μ_f from this expression be written as $m(T^f/H)$, where $m(\cdot)$ is the *markup fitting-in function*. Equilibrium market shares satisfy

$$s^f = S\left(\frac{T^f}{H}\right) \equiv \begin{cases} \frac{T^f}{H} \exp\left(-m\left(\frac{T^f}{H}\right)\right) & \text{(MNL)} \\ \frac{T^f}{H} \left(1 - \frac{1}{\sigma} m\left(\frac{T^f}{H}\right)\right)^{\sigma-1} & \text{(CES)}. \end{cases} \tag{10}$$

The system is closed with the constraint that market shares must sum to one:

$$\frac{1}{H} + \sum_{f \in \mathcal{F}} s^f = 1. \tag{11}$$

A unique solution to this system of equations is guaranteed to exist. Finally, firm-level profit

(in equilibrium) and consumer surplus can be expressed:

$$\Pi^f = \pi\left(\frac{T^f}{H}\right) \equiv \begin{cases} \frac{1}{\alpha}\left(m\left(\frac{T^f}{H}\right) - 1\right) & \text{(MNL)} \\ \frac{1}{\sigma-1}\left(m\left(\frac{T^f}{H}\right) - 1\right) & \text{(CES)} \end{cases} \tag{12}$$

and

$$CS(H) \equiv \begin{cases} \frac{1}{\alpha}\log(H) & \text{(MNL)} \\ H^{\frac{1}{\sigma-1}} & \text{(CES).} \end{cases} \tag{13}$$

Nocke and Schutz (2018, Proposition 6) establish that the markups, market shares, and profit of any firm f increase in T^f and the ratio T^f/H, but decrease in T^g for any $g \neq f$. Thus, firms that produce at lower cost, have more desirable products, and/or maintain larger product portfolios (higher T^f) or that face less competition (lower H or T^g) fare better in equilibrium.

2.1.2 Mergers and Entry in SPE

In the second stage of the game, firm F enters if it can earn positive profits in the Bertrand pricing stage, taking into account its type, T^F, its entry costs, χ, and whether a merger has occured in the first stage of the game. That is, entry occurs if the profit of firm F satisfies

$$\pi\left(\frac{T^F}{H_{*,e}}\right) - \chi \geq 0,$$

where we let $H_{*,e}$ be the market aggregator with entry, accounting for the observed merger decision of firms 1 and 2 (denoted by $*$).

In the first stage of the game, Firms 1 and 2 merge if doing so increases their combined profit in the pricing stage, taking into account the effect of the merger on the entry decision. That is, a merger occurs if and only if it increases joint profits:

$$\pi\left(\frac{T^M}{H_{m,*}}\right) \geq \pi\left(\frac{T^1}{H_{nm,*}}\right) + \pi\left(\frac{T^2}{H_{nm,*}}\right), \tag{14}$$

where $H_{m,*}$ and $H_{nm,*}$ are the aggregator with and without a merger, respectively, incorporating the best-response of the prospective entrant, and where

$$T^M = T^1 + T^2 + E \tag{15}$$

is the type of the merged firm. The term $E \geq 0$ captures the *efficiencies* generated by the merger. These may arise, e.g., through marginal cost reductions, quality improvements, broader product portfolios, or some combination of above. For simplicity, we assume that

entry occurs if and only if the merger increases the profitability of entry, even if only by an infinitesimal amount. We revisit the likelihood with which merger-induced entry occurs in Section 3.3. A unique SPE exists because the merger and entry decisions are made sequentially and a unique equilibrium exists in the pricing subgame.

2.2 Results

2.2.1 Overview

Suppose we fix a vector of underlying parameters of the model, except for (i) the magnitude E of the efficiencies generated by the merger, and (ii) the type of the prospective entrant, T^F, reflecting its competitive strength. The implicit function theorem yields a family of *neutrality curves* in (E, T^F)-space, which are plotted in Figure 1 below:

1. The **consumer surplus neutrality curve** is plotted as the dot-dash orange curve. It reflects those pairs (E, T^F) which, at our fixed parameterization, result in consumer surplus remaining unchanged between the pre-merger equilibrium and a Bertrand equilibrium featuring both merger and entry (which may or may not arise in SPE). Consumer surplus increases with the merger and entry if (E, T^F) fall above the curve, and decreases otherwise.

2. The **merger profit neutrality curve** corresponds to the solid green curve. It contains those pairs (E, T^F) under which the sum of the merging parties' profits in the ex-ante equilibrium equals the merged entity's profits in a Bertrand equilibrium, again featuring both merger and entry (which may not occur in SPE). Here, the merger is profitable in spite of entry occuring if (E, T^F) lies above this curve, and is unprofitable otherwise.

3. The **entrant profit neutrality curve** is plotted as the dashed purple line. It is composed of those pairs (E, T^F) such that the entrant's profits are constant between an equilibrium featuring both the merger and entry, and one featuring entry alone. Here, merger-induced entry is profitable for the entrant if (E, T^F) falls below the curve, and is unprofitable otherwise.

While the precise shapes and levels of these curves depend on the underlying parameterization of the model and choice of MNL or CES demand system, the qualitative features of Figure 1 remain unchanged.[13] We provide a formal statement and proof of this fact in Section 2.2.4 below.

---

[13]Versions of Figure 1 can be created numerically for any model of Bertrand competition with MNL or CES demands provided that one observes market shares. We use an MNL model with four incumbents and an outside good, each with a market share of 20 percent. Similar results obtain for alternative market shares. Appendix D describes our numerical methods.
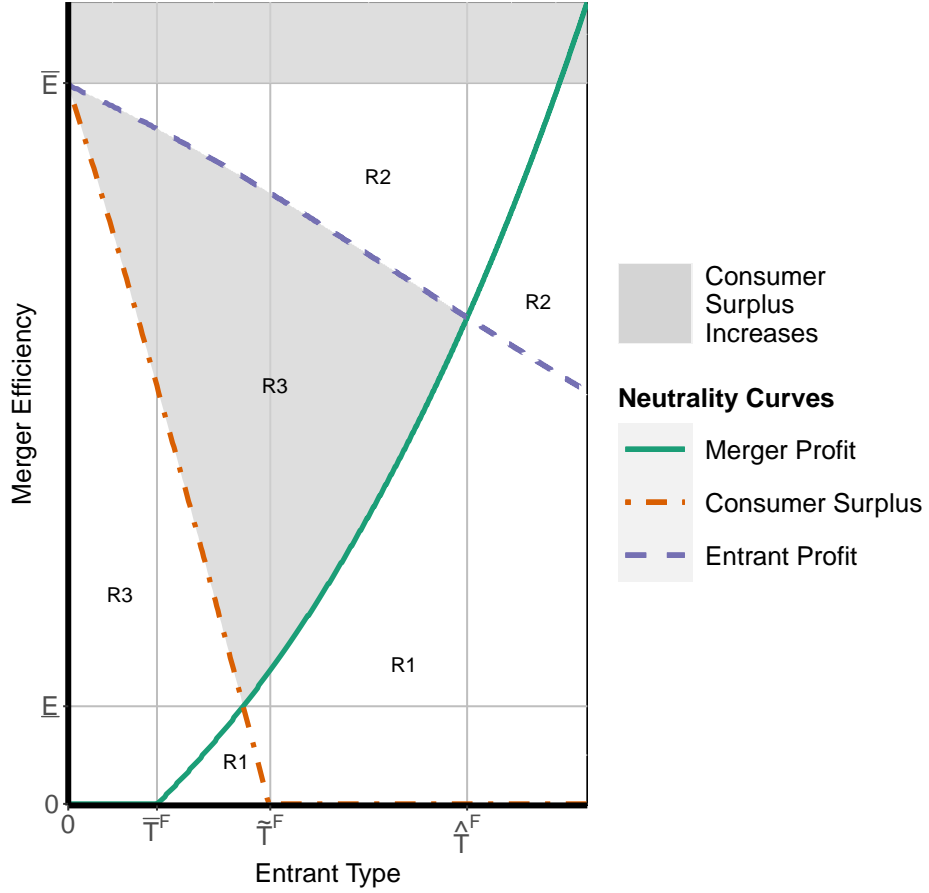
Figure 1: Merger Analysis with Entry and Efficiency

Notes: The figure illustrates the integrated framework for merger analysis with entry and efficiencies. The results are generated numerically given pre-merger market shares of 20 percent for each of four incumbents and an outside good.

2.2.2 The Compensating Entrant

The compensating efficiency of a merger is the magnitude of efficiencies such that, absent entry, the merger does not affect prices or consumer surplus (Werden 1996; Nocke and Whinston 2022). Compensating efficiencies have been used in merger review and relied upon by courts as a benchmark to determine whether predicted efficiencies are sufficient to offset a loss of competition (e.g. Miller and Sheu 2021). In Figure 1, the compensating efficiency, $\bar{E}$, corresponds to the efficiencies-axis intercept of the consumer surplus (and entrant profit) neutrality curves.

Our analysis not only recovers this concept, but additionally provides an analogous concept for entry analysis. We refer to the entrant type that, absent efficiencies, leaves consumer surplus unchanged as the *compensating entrant*. We denote this type as $\tilde{T}^F$ in Figure 1; geo-

metrically it corresponds to the type-axis intercept of the consumer surplus neutrality curve. If such a merger induces entry by a firm with a type lower than that of the compensating entrant then consumer surplus decreases, and the opposite result obtains if the entrant's type is higher than that of the compensating entrant.

Proposition 1. *Suppose that the pre-merger market shares of the merging parties, s^1 and s^2 satisfy $0 < s^1 + s^2 < 1$. Under MNL demand:*

$$\tilde{T}^F = q(s^1, s^2; H_{nm,ne}),$$

for an implicitly defined C^∞ function q depending only on the pre-merger market shares of the merging parties, and ex-ante market aggregator $H_{nm,ne}$. Under CES demand:

$$\tilde{T}^F = r(s^1, s^2;, H_{nm,ne}, \sigma),$$

where the implicit C^∞ function r also depends on σ, the elasticity of substitution.

For explicit descriptions of q and r, as well as a proof of Proposition 1, see Appendix C. As $\tilde{T}^F$ lies to the right of the type-axis intercept of the merger profit neutrality curve, we immediately obtain the following, clear-cut implication of merger induced entry absent efficiencies.

Proposition 2. *If the merger generates no efficiencies, then no SPE exists in which a merger occurs and consumer surplus does not decrease as a result.*

Absent efficiencies, mergers in our model are profitable only because they soften competition and allow firms to set higher markups. This in turn makes entry more profitable after the merger and, if entry occurs, then it benefits consumers and reduces the profit of the merged firm. Proposition 2 establishes that, absent efficiencies, if a merger induces entry at a scale sufficient to preserve consumer surplus, then the entry also is sufficient to make the merger unprofitable. Thus, without efficiencies, if a merger occurs in SPE at least one of the following must be true: (i) the type of the entrant is insufficient to preserve consumer surplus, (ii) the entry cost is large enough to deter post-merger entry, or (iii) the entry cost is small enough that entry would occur with or without the merger.

If a merger without efficiencies is profitable, then it lowers the market aggregator. Therefore, any such merger also increases incumbents' markups. As the merger does not affect marginal costs (by assumption), prices also increase. Formally,

Corollary 1. *Any profitable merger without efficiencies increases the markups and prices of incumbents, relative to a counterfactual in which the merger is prohibited.*

Notably, these results are robust to a variety of natural extensions. We could, for example, allow for multiple, heterogeneous entrants or for the monopolistically competitive fringe of entrants of Anderson et al. (2020).[14] We could also allow for *incumbent repositioning*, defined as costly investments non-merging incumbents that improve product quality, reduce marginal costs, or expand product portfolios. If a merger without efficiencies induces any such entry or incumbent repositioning at a scale sufficient to preserve pre-merger consumer surplus then the merger is not profitable.[15]

2.2.3 Equilibrium Regimes

As the underlying model parameters vary, the unique SPE of the three-stage game falls into one of three regimes: (i) no merger occurs, (ii) the merger occurs but entry does not, or (iii) both the merger and entry occur. The neutrality curves of Figure 1 show how different (E, T^F) combinations correspond to the different outcomes. Regions that yield no merger are marked with 'R1.' Regions that yield a merger without entry are marked with 'R2.' In R2, a merger with entry would increase consumer surplus, but entry does not occur in SPE. Thus, consumer surplus increases in R2 if efficiencies are such that $E > \bar{E}$ and decreases if $E < \bar{E}$. Regions that yield a merger with entry are marked with 'R3.' The gray shading shows the combinations of (E, T^F) for which the merger increases consumer surplus in SPE. The R3 regions exist for any parameterization of MNL or CES demand. Formally,

Proposition 3. *There always exist combinations of efficiencies and entrant types, (E, T^F), for which merger-induced entry (i) occurs in SPE, and (ii) induces a non-negative change in consumer surplus, relative to a counterfactual with neither the merger nor entry.*

The upper envelope of the neutrality curves for consumer surplus and merger profitability bound the efficiencies needed for merger-induced entry to preserve consumer surplus in SPE. Larger efficiencies are required for relatively low-type entrants, in order to preserve consumer surplus, but also for relatively high-type entrants, in order to maintain merger profitability. The minimum of this bound—which we refer to as the *minimum efficiency* and denote $\underline{E}$—occurs at the crossing of the neutrality functions. Any profitable merger that increases consumer surplus must have $E \geq \underline{E}$.

As in the previous subsection, we use s^1 and s^2 to refer to the merging firms' market shares in an equilibrium without the merger (or entry), and use s^M to refer to the market share of the merged firm in an equilibrium with the merger and entry.

---

[14]In comparison with Anderson et al. (2020) we also do not require free entry.

[15]Indeed, the result generalizes beyond Bertrand with MNL/CES demands to a class of aggregative games, including undifferentiated Cournot competition. We thank Volker Nocke (private communication) for demonstrating this generalization.

Proposition 4. *Under MNL demand, the minimum efficiency $\underline{E}$ is a function purely of the ex-ante market aggregator H, and pre-merger market shares s^1 and s^2:*

$$\underline{E} = H \left[s^M \exp\left(\frac{1}{1 - s^M} \right) - \sum_{i \in \{1,2\}} s^i \exp\left(\frac{1}{1 - s^i} \right) \right], \tag{16}$$

where

$$s^M = 1 - \frac{(1 - s^1)(1 - s^2)}{1 - s^1 s^2}. \tag{17}$$

Under CES demand $\underline{E}$ is additionally a function of σ, the elasticity of substitution:

$$\underline{E} = \frac{H}{(\sigma - 1)^{\sigma - 1}} \left[s^M \left(\sigma + \frac{s^M}{1 - s^M} \right)^{\sigma - 1} - \sum_{i \in \{1,2\}} s^i \left(\sigma + \frac{s^i}{1 - s^i} \right)^{\sigma - 1} \right], \tag{18}$$

where

$$s^M = \frac{\sigma}{\sigma - 1} \left(1 - \frac{\left(1 - \frac{\sigma - 1}{\sigma} s^1 \right) \left(1 - \frac{\sigma - 1}{\sigma} s^2 \right)}{1 - \frac{\sigma - 1}{\sigma} s^1 s^2} \right). \tag{19}$$

Figure 1 also shows regions in which merger-induced entry does not occur. One interesting case is when $T^F > \widehat{T}^F$, the entrant type at the crossing of the merger and entrant profit neutrality curves. There, large efficiencies are necessary to maintain merger profitability given the strength of the entrant. However, efficiencies of this magnitude render *entry* for firm F unprofitable. Thus, in this regime, if the merger occurs, then merger-induced entry does not. A similar outcome obtains in the event that efficiencies fall in the goldilocks region between the compensating efficiency $\bar{E}$ and the entrant profit neutrality curve, as the efficiencies are too small to preserve consumer surplus, but sufficiently large to render merger-induced entry unprofitable. Thus, because of subgame perfect reasoning on the part of firms and the interaction with entry, consumer welfare is *non-monotone* in the level of efficiencies.

2.2.4 A Structural Characterization

The following results establish that all of the qualitative features of the model highlighted above obtain under any calibration of the model. Thus they provide a complete characterization of the equilibrium outcomes of our model. The formal statements below correspond to the case of MNL demands, however analogous results hold, *mutatis mutandis*, for CES demands as well.

Proposition 5 (Welfare Neutrality Characterization). *For any $0 \leq T^F \leq \tilde{T}^F$, there exists a unique efficiency level $E = \Psi(T^F)$ such that the merger and entry leave consumer surplus unchanged relative to a no-merger, no-entry counterfactual. This function Ψ satisfies:*

(i) *For all $0 \leq T^F \leq \tilde{T}^F$,*

$$\frac{d\Psi}{dT^F} < 0.$$

(ii) *The compensating efficiency is obtained as $\bar{E} = \Psi(0)$, and $\Psi\big(\tilde{T}^F\big) = 0$.*

(iii) *For any parameterization such that $E = \Psi(T^F)$, the combined pre-merger market shares of the merging parties equals the combined post-merger market shares of the merged firm and the entrant.*

Proposition 5 shows that no matter the parameterization, the qualitative features of the welfare neutrality curve remain the same: its efficiencies-intercept is the compensating efficiency, and it is downward sloping as a function of the entrant's type. Notably, along this curve it also highlights that the pre- and post-merger and entry market shares satisfy an 'adding up' condition $s^1 + s^2 = s^M + s^F$. Conditional on leaving consumer welfare unchanged, the only effect of entry is to cannibalize the merged firm's market share.

Proposition 6 (Merger Profit Neutrality Characterization). *For any $\bar{T}^F \leq T^F$, there exists a unique efficiency level $E = \Phi(T^F)$ such that the merged firm's profits, in an equilibrium featuring entry, equals the sum of the merging firms' pre-merger profits. The function Φ satisfies:*

(i) *For all $T^F \geq \bar{T}^F$,*
$$\frac{d\Phi}{dT^F} > 0.$$

(ii) *There exists some entrant type T^F such that $\bar{E} = \Phi(T^F)$, and $\Phi(\bar{T}^F) = 0$.*

(iii) *For any parameterization such that $E = \Phi(T^F)$, the merged firm's market share s^M post-entry is given by:*
$$s^M = 1 - \frac{(1-s^1)(1-s^2)}{1 - s^1 s^2},$$
where s^1 and s^2 are the merging parties' pre-merger market shares.

Proposition 6 shows that the merger profit neutrality curve is always upward sloping in (T^F, E)-space, and always crosses the compensating efficiencies line. Thus it is possible for both a merger to generate compensating efficiencies, while simultaneously attracting sufficient entry to render the merger profit-neutral. It also establishes that, along this neutrality locus, the merged firm's market share is a function purely of the merging parties' ex ante equilibrium market shares.

Proposition 7 (Entrant Profit Neutrality Characterization). *For any $T^F > 0$, there exists a unique efficiency level $E = \Theta(T^F)$ such that the entrant would obtain the same profit, conditional upon entry, whether or not the merger occurs.[16] The function Θ satisfies:*

---

[16] If $T^F = 0$ then the entrant earns zero profit conditional upon entry, regardless of the efficiencies of the merger.

(i) *For all $T^F > 0$,*

$$\frac{d\Theta}{dT^F} < 0.$$

(ii) *As T^F approaches 0 from above,*

$$\lim_{T^F \to 0^+} \Theta(T^F) = \bar{E}.$$

   Moreover, $\Theta(T^F) > 0$ for all $T^F > 0$.

(iii) *For any parameterization such that $E = \Theta(T^F)$, conditional upon entry, the merger does not affect consumer surplus. Moreover, entry always strictly increases consumer surplus, hence $\Theta > \Psi$ pointwise.*

Finally, Proposition 7 provides a characterization of the entrant's profit neutrality curve. It always slopes downward in (E, T^F) space and always lies strictly above the consumer surplus neutrality curve. Together these results guarantee that there is an open set of parameters under which merger-induced entry occurs and consumers benefit.

3   Extensions

In this section, we consider a variety of generalizations of our base model. We find that the same qualitative insights obtain, suggesting that the practical take-aways of our model are robust to the particular specifications used to model competition.

3.1   Nested Logit and Nested CES

While a natural benchmark, and commonly used in antitrust practice (e.g. Werden and Froeb 1994, 2002), MNL and CES demands feature specific substitution patterns between products. In this section, we consider the cases of nested MNL and CES (NMNL and NCES) demands, in order to study the robustness of our results to markets with richer structures.[17] This may be of particular interest in settings for which a known prospective entrant would be either a close or distant competitor to the merging firms.

In nested demand models, products are grouped into exhaustive and mutually exclusive sets, or "nests," with products in the same nest being close substitutes and products in different nests being distant substitutes. Let each product $j \in \mathcal{J}$ belong to a nest, $g(j) \in \mathcal{G}$, and let the set of products in nest g be $\mathcal{J}_g$. We assume that there is an additional nest ($g = 0$), that contains only the outside good ($j = 0$).

---

[17]For recent examples of NMNL/NCES demands in antitrust practice, see the 2016 Aetna/Humana merger trial (Bayot et al., 2022), or the 2022 trial of the "Northeast Alliance" of American Airways and JetBlue.

With NMNL demand, each consumer purchases a single product $j \in \mathcal{J}$ or forgoes a purchase by selecting the outside good ($j = 0$). The indirect utility that consumer i receives from product $j \in \mathcal{J}$ in nest $g(j)$ is

$$u_{ij} = v_j - \alpha p_j + \zeta_{ig(j)} + (1 - \rho)\epsilon_{ij},$$

where ϵ_{ij} is iid Type I extreme value and $\zeta_{ig(j)}$ has the unique distribution such that $\zeta_{ig(j)} + (1 - \rho)\epsilon_{ij}$ is also iid Type I extreme value (Berry 1994; Cardell 1997). The nesting parameter, $\rho \in [0, 1)$, characterizes the correlation in preferences for products of the same nest; larger values correspond to more substitution within nests, and less substitution between nests. With $\rho = 0$ the model collapses to MNL demand. Market shares are given by

$$s_j(p) = \frac{\exp(v_j - \alpha p_j)}{\sum_{k \in g(j)} \exp(v_k - \alpha p_k)} \frac{\left(\sum_{k \in g(j)} \exp(v_k - \alpha p_k)\right)^{1-\rho}}{1 + \sum_{g \in \mathcal{G}} \left(\sum_{k \in g} \exp(v_k - \alpha p_k)\right)^{1-\rho}}, \tag{20}$$

where the first ratio is the share of product j within its nest and the second ratio is the combined share all products $k \in g(j)$.

With NCES demand, consumer utility takes the form

$$u_i = \left(\sum_{g \in \mathcal{G}} Q_g^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}},$$

where

$$Q_g = \left(\sum_{j \in \mathcal{J}_g} v_j^{\frac{1}{\sigma}} q_j^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

and $\sigma \geq \gamma > 1$ are the elasticities of substitution between products in each of the nests. If $\sigma = \gamma$, this collapses to the CES model. Market shares are given by

$$s_j(p) = \frac{v_j p_j^{1-\sigma}}{\left(\sum_{k \in \mathcal{J}_{g(j)}} v_k p_k^{1-\sigma}\right)^{\frac{\sigma-\gamma}{\sigma-1}} \left(1 + \sum_{g \in \mathcal{G}} \left(\sum_{k \in \mathcal{J}_g} v_k p_k^{1-\sigma}\right)^{\frac{\gamma-1}{\sigma-1}}\right)}. \tag{21}$$

In Bertrand equilibrium, both NMNL and NCES demand exhibit the type aggregation and common markup properties if all products of any given firm are located in a single nest (Nocke and Schutz (2019)). We focus attention on that case in order to obtain analytic results. The

firm types take the form

$$T^f \equiv \begin{cases} \sum_{j \in \mathcal{J}^f} \exp\left(\frac{v_j - \alpha c_j}{1-\rho}\right) & \text{(NMNL)} \\ \sum_{j \in \mathcal{J}^f} v_j c_j^{1-\sigma} & \text{(NCES)} \end{cases} \tag{22}$$

and thus firms with lower costs, more desirable products, and broader product portfolios have larger types. The ι-markups are defined as

$$\mu^f \equiv \begin{cases} \frac{\alpha}{1-\rho}(p_j - c_j) & \forall j \in \mathcal{J}^f & \text{(NMNL)} \\ \sigma \frac{p_j - c_j}{p_j} & \forall j \in \mathcal{J}^f & \text{(NCES)}. \end{cases} \tag{23}$$

The Bertrand equilibrium can be characterized as a vector of ι-markups, $\{\mu^f\} \, \forall f \in \mathcal{F}$, a vector of firm-level market shares, $\{s^f\} \, \forall f \in \mathcal{F}$, a vector of nest-level aggregators $\{H_g\} \, \forall g \in \mathcal{G}$, and a market aggregator, H. As substantial notation is required, we defer these characterizations to Appendix A.

3.1.1 Results

For ease of exposition, we focus on the case of zero efficiencies. Our first result shows that Proposition 2 generalizes straightforwardly to the case of NMNL or NCES demands, when the merging firms and the entrant have products in the same nest.

Proposition 8. *Suppose the merger generates no efficiencies, and the products of the merging firms and the entrant belong to the same nest. Then no SPE exists in which a merger occurs and consumer surplus does not decrease as a result.*

Thus we once again find that profitable mergers without efficiencies are incompatible with merger-induced entry sufficient to preserve consumer surplus, so long as the product lines of the merging firms and the entrant are close enough substitutes.

A technical advantage of the revealed preference approach underpinning Propositions 2 and 8, however, is that it remains valid under mild perturbations of the underlying demand framework. We leverage this to obtain a 'robust' analogue of Proposition 8, allowing for entry into an *arbitrary* nest, provided the appropriate nesting parameter (ρ in the case of NMNL, σ for NCES) is not too large.

Formally, this requires us to first establish the continuity of the unique Bertrand pricing equilibrium as a function of the relevant nesting parameter in a neighborhood of the value for which the demand system collapses to its non-nested counterpart ($\rho = 0$ for NMNL, $\sigma = \gamma$ for NCES). To the best of our knowledge, this technical result is new to the literature.[18] We establish:

---

[18]Moreover, due to our need to consider parameter values that lie on the 'boundary' of their respective ranges,

Proposition 9. *For any fixed vector of model primitives, the mapping taking ρ (resp. σ) to the unique Bertrand equilibrium of the pricing game with NMNL (resp. NCES) demand is continuous on a neighborhood of 0 (resp. γ).*

By appeal to Proposition 9, we obtain a "robust" analog of Proposition 8 that does not depend on whether the merging firms' and the prospective entrant's product lines belong to the same nest.

Proposition 10. *Suppose the merger generates no efficiencies. Under NMNL (resp. NCES) demand, there exists $\bar{\rho} > 0$ (resp. $\bar{\sigma} > \gamma$) such that, for any $\rho \leq \bar{\rho}$ (resp. $\sigma \leq \bar{\sigma}$), no SPE exists in which a merger occurs and consumer surplus does not decrease as a result.*

We use numerical simulations to investigate how large the nesting parameter must be in order to generate an SPE with a merger and merger-induced entry that is sufficient to preserve consumer surplus, i.e. how large $\bar{\rho}$ (resp. $\bar{\sigma}$) are in Proposition 10. For concreteness, we focus on the case of NMNL demand and consider settings with four, six, and eight incumbents of equal market share, evenly split between two nests, and with an outside good that has a 20% market share. We calibrate incumbent types using the market shares, for values $0 \leq \rho \leq 0.9$. We then simulate a merger between two incumbents under the assumption that the merging firms' products are in one nest and the entrants' products are in the other nest, and consider entrant types in the range $0 \leq T^F \leq \frac{3}{2}T^1$.

Figure 2 plots the results for the case of six incumbents. The vertical axis is ρ (the "nesting parameter") and the horizontal axis is T^F/T^1 (the "entrant type ratio"). A merger that induces entry is profitable for combinations of the nesting parameter and the entrant type ratio that fall above the solid blue neutrality curve, and increases consumer surplus for combinations that fall below the dashed orange neutrality curve. Thus, the shaded gray area between these curves provides the region for which a merger could increase consumer surplus in SPE. The minimum ρ under which consumer surplus increases is 0.446. Qualitatively similar results obtain in the case of four or eight incumbents.

It is useful to interpret the nesting parameter ρ in terms of its implications for the diversion that arises between the merged firm and the entrant. With NMNL demand, the diversion from one product k to another product j when the products are in different nests is given by:

$$DIV_{k \to j} = (1 - \rho) \frac{s_j}{1 - s_k - \rho(\bar{s}_{k|g} - s_k)}, \tag{24}$$

which is decreasing in ρ. Here, $\bar{s}_{k|g}$ denotes the market share of product k conditional on its nest being selected (Berry, 1994).[19] In the post-merger, post-entry equilibrium with the

---

the standard implicit function theorem is not applicable, necessitating a slightly more delicate approach. See Appendix C for details.

[19]In other words, $\bar{s}_{k|g} = s_k / \sum_{m \in \mathcal{J}_{g(k)}} s_m$.
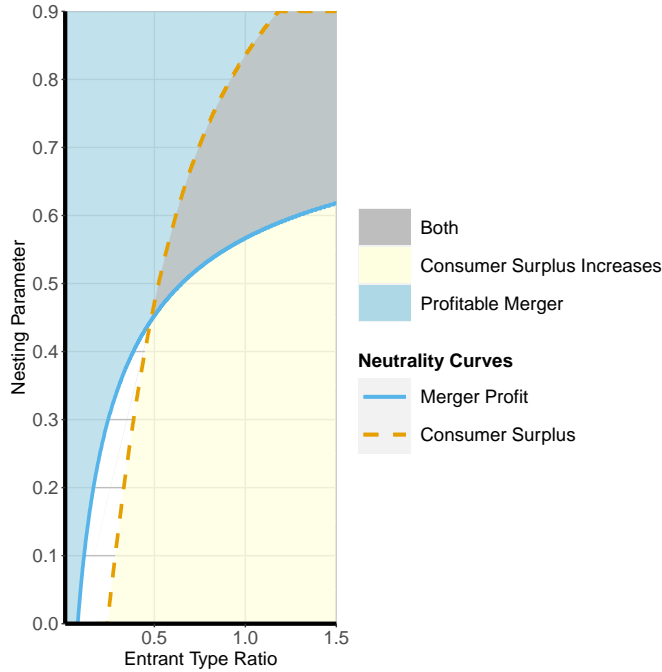
Figure 2: Numerical Analysis of Mergers and Entry with NMNL Demand

Notes: The plot shows the nesting parameters (ρ) and entrant type ratios (T^F/T^1) for which a merger with entry increases consumer surplus (shaded yellow), increases the merging firms' profit (shaded blue), or both (shaded gray). The corresponding neutrality curves for merger profitability and consumer surplus are plotted as solid blue and dashed orange lines, respectively.

smallest nesting parameter where consumers benefit ($\rho = 0.446$), diversion from the merging firms to the entrant is 5.6%. Holding the equilibrium shares constant, the same diversion ratio with a non-nested MNL model ($\rho = 0$) is 9.8%. Therefore, the least distant entrant that eliminates the consumer surplus loss of a profitable merger captures diversion that is little more than half of what it would obtain under MNL demand. Thus, absent efficiencies, a prospective entrant must be substantially differentiated from the merging firms to eliminate consumer surplus loss in SPE.

Heuristically, one might expect such scenarios to be unlikely in practice. In light of the above, a necessary condition for consumer surplus-enhancing merger-induced entry is for the products of the entrant to be substantially differentiated from those of the merging firms.[20] The entrant's product line must fall into the region where consumers who prefer the entrant's products gain from more variety, but the entrant does not steal too much market share from the merging firms. However, if the entrant is a sufficiently distant competitor (i.e. $\rho \gg \bar{\rho}$), then the effect of the merger on the profitability of entry may be small. Thus, in markets where merger-induced entry could be a viable remedy for an otherwise anticompetitive

---

[20]This is also sufficient, conditional on merger-induced entry occurring.

merger, the likelihood of such entry may be lower, because the cost of entry may need to fall within a narrower range. We revisit this point more formally in Section 3.3.

3.2 Random Coefficients Logit

The random coefficients logit (RCL) demand system is widely employed in modern empirical studies due to its extreme flexibility. In this section, we explore whether the basic intuition that emerges from our analysis of NMNL and NCES demand—that entry by a distant competitor can offset the adverse effect of a profitable merger in SPE—extends to the RCL model. Throughout this section we rely on numerical analysis and an empirical exercise due to the RCL model's failure to exhibit the type aggregation or common markup properties underpinning our earlier analytic results.

We first consider whether merger-induced entry sufficient to mitigate the welfare from a profitable merger (without efficiencies) requires an entrant with products that are distant substitutes to those of the merging firms. We assume the indirect utility that consumer i receives from product j:

$$u_{ij} = (1 + \beta_i)v_j - \alpha p_j + \epsilon_{ij}, \tag{25}$$

where ϵ_{ij} is iid Type I extreme value and $\beta_i \sim N(0,1)$ is a consumer-specific valuation for quality. There are two single-product incumbents, each with $v_j = 4$ and $c_j = 2$. We consider four values of the price parameter: $\alpha = (1, 2, 3, 4)$. The larger values imply more elastic demand. With $\alpha = 4$, the pre-merger equilibrium features prices of 2.36, incumbent market shares of 6.4%, and a diversion ratio between incumbents of 45%. With $\alpha = 1$, these statistics are 3.72, 30%, and 72%, respectively. We consider entrants with marginal costs and qualities that range between -2 and 8. With a step size of 0.05, this yields 40,401 entrants. We simulate a merger between the incumbents under the assumption that it induces entry by one of the entrants. Iterating through the entrants, we determine whether consumer surplus and the merging firms' profit increase relative to the pre-merger baseline.

Figure 3 summarizes the results. In each panel, the shaded gray region provides the entrant qualities and marginal costs for which the merger is both profitable and increases consumer surplus. In the top left panel ($\alpha = 4$), this region features entrant marginal costs that are close to zero or negative and entrant quality that is substantially less than that of the merging firms.[21] Comparing across panels, as demand becomes less elastic and incumbent market powers grows, the gray region requires even lower entrant marginal costs and qualities. In the bottom right panel ($\alpha = 1$), the region does not exist within the considered marginal cost and quality ranges.

---

[21] We suspect that a similar region exists for entrant costs and quality that are both much higher than the merging firms, but computing equilibrium in that parameter range is difficult for numerical reasons.
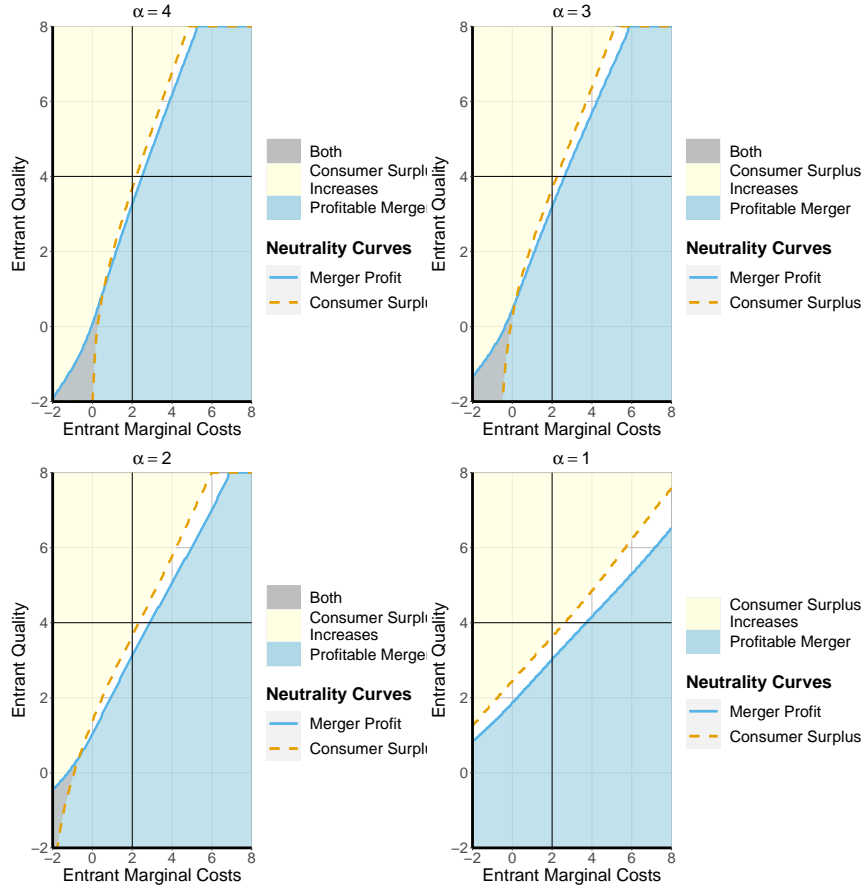
Figure 3: Numerical Results for RCL Demand with $\alpha = (4, 3, 2, 1)$

Notes: The panels show the combinations of entrant quality and marginal cost for which a merger with entry increases consumer surplus (shaded yellow), increases the merging firms' profit (shaded blue), or both (shaded gray). The corresponding neutrality curves for merger profitability and consumer surplus are plotted as solid blue and dashed orange lines, respectively. The marginal cost and quality of the merging firms are plotted with the black vertical and horizontal lines.

We interpret these results as indicating that the intuition behind our results for the NMNL and NCES models extends to the RCL model: merger-induced entry sufficient to preserve consumer surplus can be compatible with a profitable merger, but only if the entrant's products are differentiated enough from those of the merging firms.[22] The model also is informative of the entrant characteristics under which surplus-preserving merger-induced entry can arise in SPE. Thus in empirical work, knowledge of the production technologies could be paired with the model to determine whether merger-induced entry that restores consumer surplus in SPE is plausible. For example, the model might indicate that the entrant's marginal costs would have to be negative, or that its quality would have to be much higher than that of the

---

[22]See also the discussion after Figure 2

incumbents.

3.2.1 Example: Surplus-Preserving Entry in the US Beer Industry

We illustrate the types of inferences that can be drawn in practice using the demand estimates of Miller and Weinberg (2017) on the US beer industry. The data include monthly and quarterly observations from 2005 to 2011 on prices and quantities of the 39 products sold by the major brewers (13 flagship brands $\times$ 3 package-size categories) in each of 39 distinct geographic regions.[23] The indirect utility that consumer i in region r and period t receives from product j is given by

$$u_{ijrt} = x'_j \beta^*_i + \alpha^*_i p_{jrt} + \sigma^D_j + \tau^D_t + \xi_{jrt} + \epsilon_{ijrt},$$

where x_j is a vector of non-price characteristics and (σ^D_j, τ^D_t) are product and time fixed effects. We restrict attention to Boston in the second quarter of 2008.[24] We use the RCNL-2 specification, in which the random coefficients (β^*_i, α^*_i) allow income to affect preferences for price, package size, and calories. The quality of product j and period t can be defined as

$$v_{ij} \equiv \sigma^D_j + \tau^D_t + \xi_{jt}.$$

We recover product qualities using the demand estimates and marginal costs from the first order conditions implied by Bertrand competition. We refer readers to Miller and Weinberg (2017) for details on the market and the demand system.

We simulate the Miller/Coors merger with a series of entrants that differ in their marginal cost and quality.[25] We assume that all entrants have the same product portfolio as Coors (e.g., as Coors sells a "Light 12 pack," so too does the entrant). However, we allow the entrant's marginal costs to be up to 50% lower than those of Coors, or up to 100% higher. We also allow for the entrant's qualities to be substantially higher than the highest-quality product in the market, or substantially lower than the lowest-quality product. Specifically, letting the maximum and minimum observed product quality be $\overline{v}$ and $\underline{v}$, we consider entrant qualities between $\underline{v} - (\overline{v} - \underline{v})$ and $\overline{v} + (\overline{v} - \underline{v})$. We assume that marginal costs and qualities outside these ranges are infeasible given the existing production technology.

Figure 4 shows the results. The vertical axis provides the entrant's quality and the hori-

---

[23]These data are constructed from the IRI Marketing Data Set (Bronnenberg et al. (2008)).

[24]This reduces computational time. Boston is the first region alphabetically, and the second quarter of 2008 is the last full quarter before the consummation of the Miller/Coors merger (which was structured as a joint venture).

[25]Our analysis focuses on how mergers and entry can be evaluated with RCL demand, so we do not consider the efficiencies and coordinated effects of the merger. Nocke and Whinston (2022) take a similar approach in using the data to explore the connection between HHI and merger price effects.
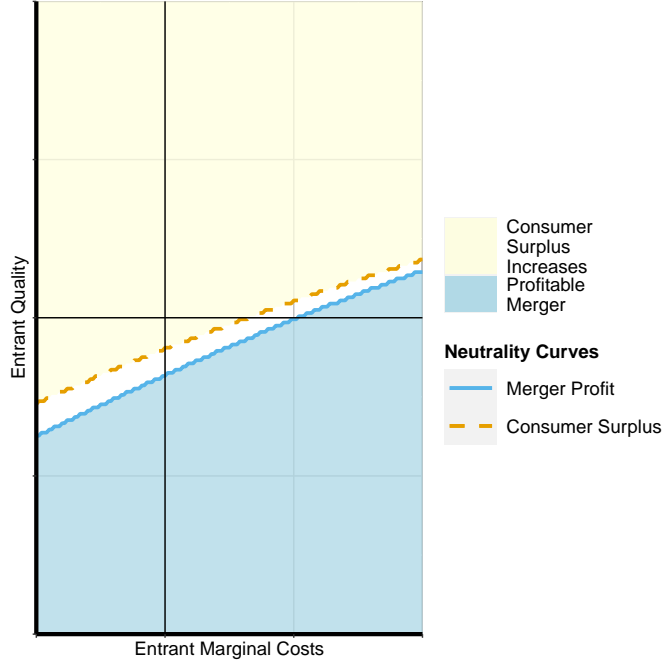
Figure 4: Entry Analysis for the Miller/Coors Merger in the Beer Industry

Notes: The plot shows the combinations of entrant quality and marginal cost for which a merger with entry increases consumer surplus (shaded yellow) and increases the merging firms' profit (shaded blue). The corresponding neutrality curves for merger profitability and consumer surplus are plotted as solid blue and dashed orange lines, respectively. The marginal cost and quality of Coors, the smaller of the two merging firms, are plotted with the black vertical and horizontal lines.

zontal axis provides its marginal cost. As shown, there is no overlap between the region in which consumer welfare increases due to the merger (shaded yellow) and the region in which the merger increases the profit of the merging firms (shaded blue). Therefore, in this context we find that merger-induced entry sufficient to preserve consumer surplus is incompatible with merger profitability, absent efficiencies. In this case, a revealed preference on the part of the merging firms indicates that barriers may prevent entry that restores the pre-merger level of consumer surplus.

3.3 Likelihood of Entry

The results of Section 2 establish that a merger increases the profitability of entry if merger efficiencies are sufficiently small. Under the assumption of fixed costs of entry $\chi \geq 0$, merger-induced entry occurs in SPE if and only if:

$$\pi \left(\frac{T^F}{H_{nm,e}} \right) < \chi \leq \pi \left(\frac{T^F}{H_{m,e}} \right). \tag{26}$$

In particular, the entry cost must be large enough to deter entry without the merger, but small enough that entry with the merger is profitable.[26] This suggests a bounds approach for the evaluation of merger-induced entry (e.g. Li et al. 2022; Ciliberto et al. 2021; Fan and Yang 2023).

Figure 5 summarizes the bounds for the case of MNL demand and four symmetric incumbents (and an outside good) each with a market share of 20 percent. The left panel plots the profit that the entrant would receive with and without a merger as a function of the entrant's type. Both lines are upward-sloping, as higher-type entrants obtain higher profit. For any entrant type, profit is higher with the merger than without. The dashed blue line in the right panel shows the corresponding percentage change in the entrant's profit due to the merger. For the compensating entrant, located at the second vertical line, a merger increases profit from 0.079 to 0.087, indicating that merger-induced entry would occur only if χ falls within this range. The solid orange line in the right panel corresponds to a merger with lower bound efficiencies $\underline{E}$ (see Proposition 4), and shows that the implied bounds on the entry cost are tighter. This occurs because greater efficiencies make the post-merger environment less attractive for the entrant, all else equal.

To obtain results valid across market structures, we calibrate a model of Bertrand competition with MNL demands for randomly-drawn market share configurations. We consider cases with two, three, four, and five incumbents separately, and assume an outside good market share of 20 percent.[27] For each market share configuration, we consider a merger between two arbitrarily-selected incumbents, obtain the type of the compensating entrant, and then use simulations to determine how much the merger increases the profitability of entry.

Figure 6 summarizes the results. The panels correspond to the number of incumbents, the horizontal axes show the change in the HHI caused by the merger (not accounting for entry), and the vertical axis is the percentage increase in the entrant's profit due to the merger. For the purpose of the figure, we obtain the change in HHI as $\Delta\text{HHI} \equiv 2s_1s_2$. The 2023 US Merger Guidelines rely on ΔHHI as an indicator of likely market power effects; Nocke and Schutz (2019) show that it indeed provides a good proxy in the context of Bertrand competition and MNL demand. We find a highly linear relationship between ΔHHI and the effect of merger on the profitability of entry. While mergers that create bigger changes in concentration increase the profitability of compensating entry to a greater extent, we find that for more than two incumbents, every 500 points of ΔHHI almost uniformly provides a 6%-7% profit increase for the compensating entrant. For a given ΔHHI, the profit effect is more pronounced the fewer

---

[26]In an empirical application, it may be appropriate to decompose χ into an upfront entry cost (EC) and the present value of fixed costs (FC), such that $\chi \equiv (1-\delta)EC + FC$, where δ is the discount rate. The role of the discount rate is greater, the larger are upfront entry costs relative to fixed costs, all else equal.

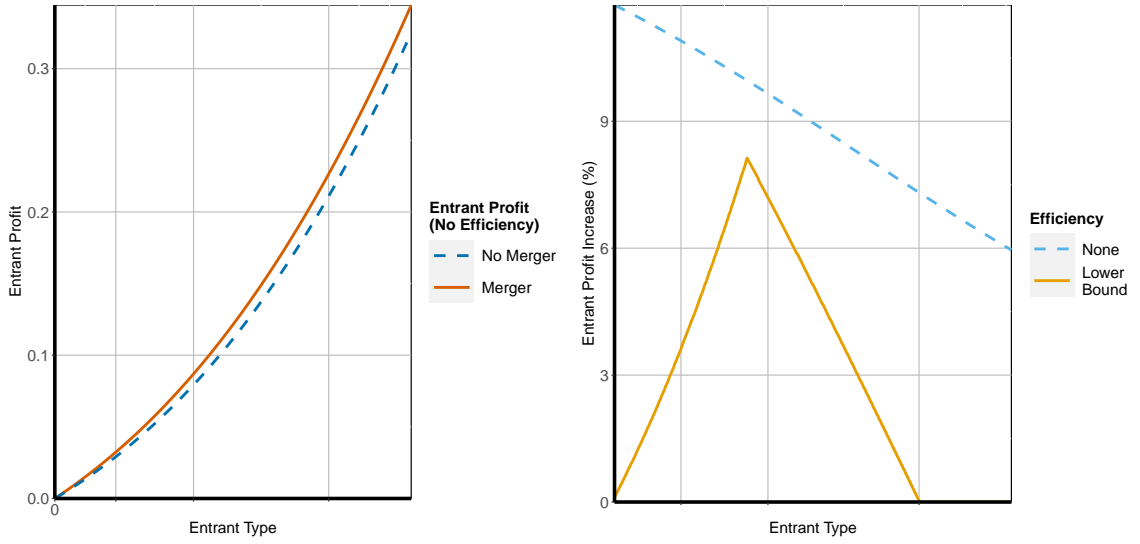[27]This outside good choice does not affect our conclusions.

Figure 5: The Entrant's Profit Opportunity

Notes: The left panel plots the profit that the prospective entrant would receive both without and with the merger, assuming no merger efficiencies. The right panel plots the percentage change in entrant profit due to a merger without efficiencies, along with the analogous percentage change due to a merger with lower bound efficiencies. The figures are generated numerically for a market with four incumbents and an initial outside good share of 20 percent.

the number of incumbents, particularly in cases of three vesus two incumbents.[28] Overall, we find that ΔHHI provides simple, practical, and exogenous bounds on the scope for mergers to increase the profitability of entry.

If one takes a stance on the empirical distributions of entry costs then the inequalities in (26) inform the probability with which entry occurs. Tighter bounds obtain if the entrant would be a distant competitor of the merging firms. The reason is that a merger increases the profitability of entry to the extent that the merging firms raise price and thereby induce more consumers to select the entrant's product. If relatively fewer consumers switch to the entrant, the effect of the merger on the entrant's profit is smaller, all else equal. This can be seen formally by differentiating the entrant's profit function with respect to the price of one the merging firms. With a few algebraic steps, we obtain that this derivative is proportional to the diversion between the merging firm and the entrant:

$$\frac{\partial \pi^F}{\partial p^1} = s^1 \frac{\mu^F}{\mu^1} DIV_{1 \to F}. \qquad (27)$$

---

[28]With two incumbents, the merging firms have a combined share of 80 percent by construction, so variation in ΔHHI comes exclusively from how asymmetric their market shares are.
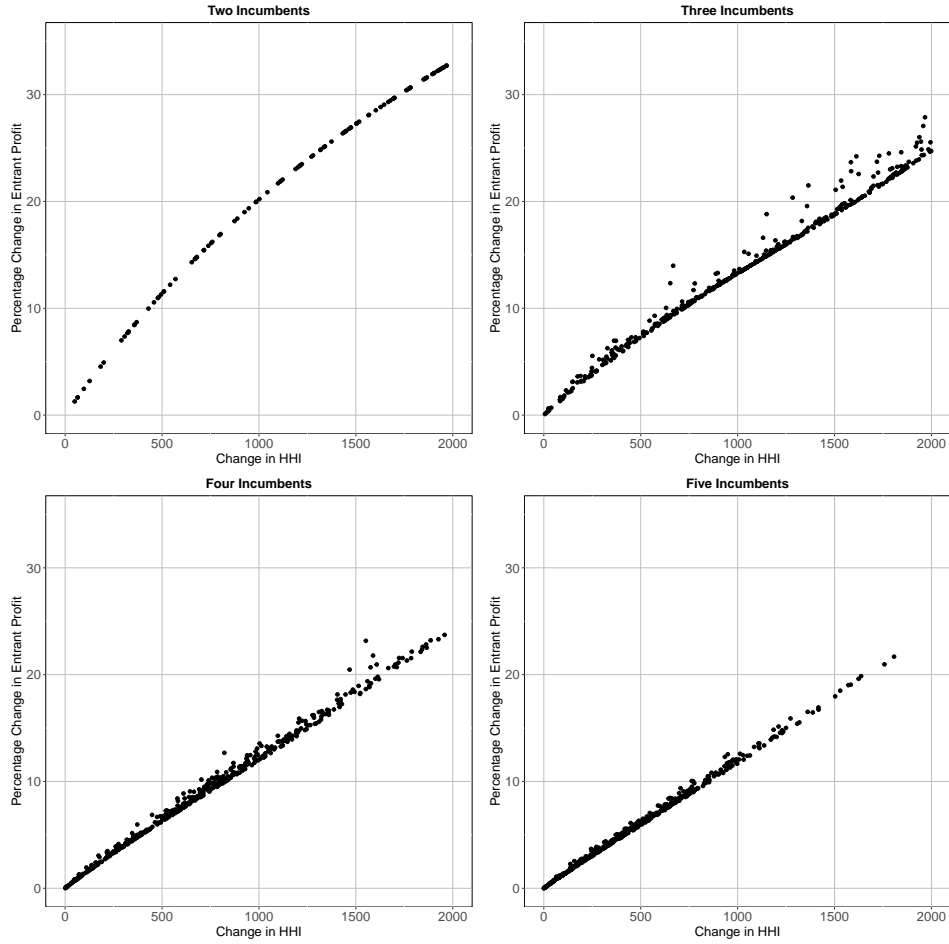
Figure 6: Effect of a Merger on Entrant Variable Profit

Notes: The panels correspond to models with two, three, four, and five incumbents, respectively. To produce each dot, we randomly draw market shares for the incumbents, consider a merger between two of them, calculate the type of the compensating entrant, and determine how much more profitable entry would be for that entrant due to the merger.

Earlier we developed that, for mergers without efficiencies, merger-induced entry sufficient to preserve consumer surplus in SPE requires that the entrant be a distant competitor of the merging firms. Equation (27) provides a formal basis to think that the more distant is the entrant, the narrower is the scope for profitable merger-induced entry.

4   Application to T-Mobile/Sprint

We apply the empirical frameworks to the T-Mobile/Sprint merger, which combined two of the four US national providers of mobile wireless telecommunications service. The Depart-

ment of Justice and the Federal Communications Commission (FCC) approved the merger conditional on certain behavioral remedies and the divestiture of Boost—a Sprint prepaid brand—to DISH, a prospective entrant. The merger then was challenged unsuccessfully in Federal District Court by several states. The Court's decision addressed whether adverse competitive effects from the loss of competition would be offset by DISH's entry.

We analyze the merger using a Bertrand MNL model of competition among the four national providers: Sprint, T-Mobile, Verizon, and AT&T. We calibrate the model using publicly available data on subscriber shares, prices, and markups. We also use a market elasticity of demand that appears in regulatory filings.[29] While model calibration and merger simulation are often used by the Agencies to assess the competitive effects of mergers, to our knowledge the sort of entry analysis we employ here has not been used so far in practice (at least prior to our presentations of this research to the Agencies).

These data are sufficient to generate the graphs that we developed earlier in the paper. In Figure 7, starting with the top panel, we obtain efficiencies bounds ($\underline{E}$ and $\bar{E}$) that are equivalent to marginal cost reductions of 1.6% and 4.0%, respectively. Efficiencies must be inside these bounds to generate a pro-competitive merger with induced entry. If the true efficiency is less than the lower bound then the merger harms consumers, and if it is greater than the upper bound then merger-induced entry does not occur (though consumers benefit overall from the merger). The bottom two panels characterize the effect of the merger on the profitability of entry. Without efficiencies, the merger increases the profitability of any entrant by less than 7%. With the lower bound efficiencies necessary to ensure merger profitability, an entrant's profit increases by well less.

We now focus specifically on DISH, the prospective entrant identified in the course of litigation. At the time of the merger, DISH had already acquired a substantial portfolio of spectrum licenses in FCC auctions, so it owned at least some of the market-specific assets necessary to compete as an independent firm. The Court's decision states that:

> DISH is well positioned to become a fourth [mobile network operator] in the market, and its extensive preparations and regulatory remedies indicate that it can sufficiently replace Sprint's competitive impact....[30]

We interpret this language as indicating a belief that DISH could offer service at a quality and cost that are similar to Sprint. As our calibration obtains a Sprint type of $T^1 = 5.00$,

---

[29]Details on the data and calibration process are provided in Appendix D. The market elasticity of demand is the percentage change in the (combined) share of the inside products due to a one percent change in their weighted-average price. Letting ϵ be the market elasticity of demand, with MNL demand we have $\epsilon = -\alpha s_0 \bar{p}$, where $\bar{p}$ is the weighted-average price. In calibration, the market elasticity determines the the outside good's share, which is not observed in the data.
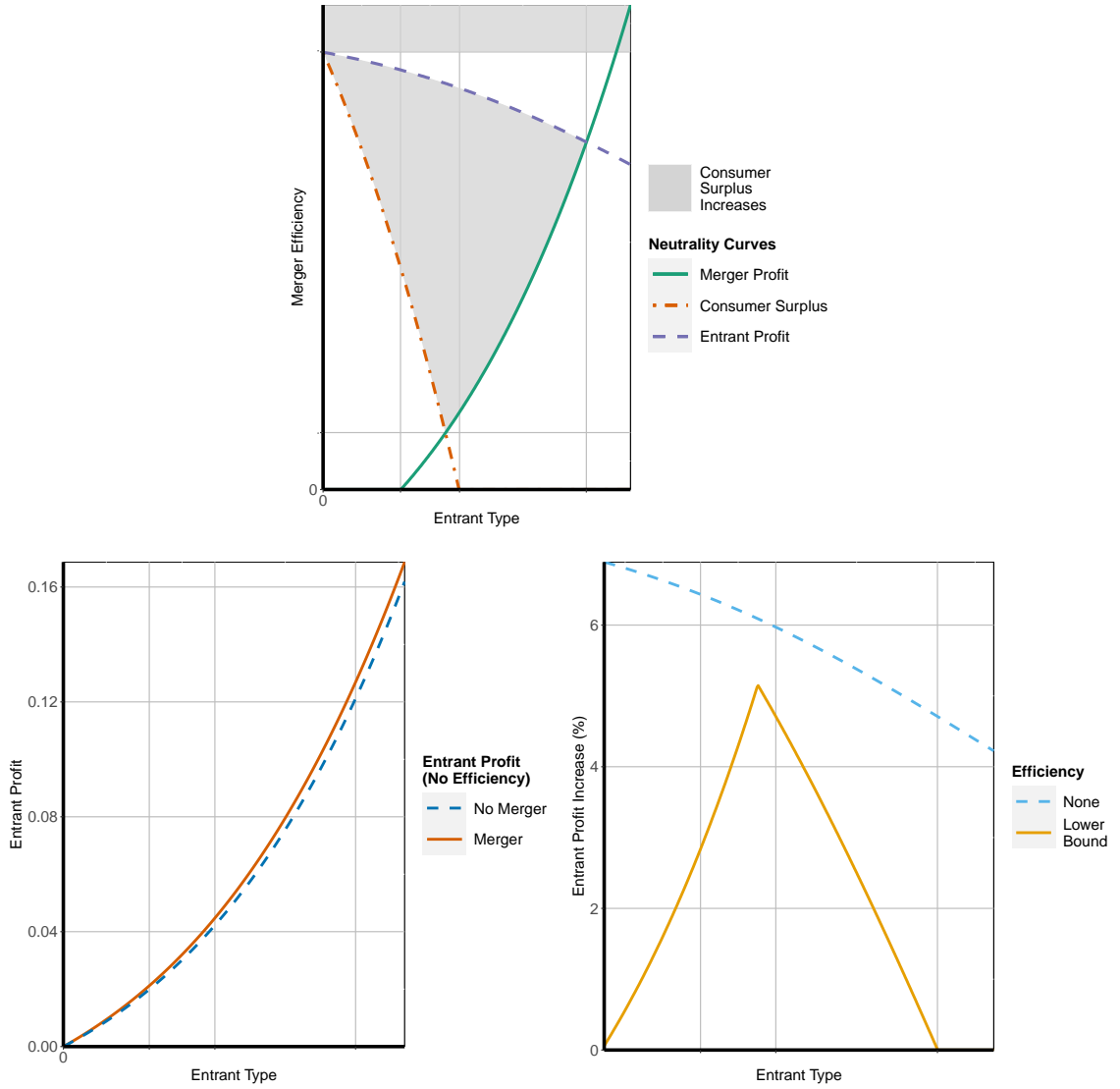
[30]See p. 117 of the opinion.

Figure 7: Application to T-Mobile/Sprint Merger

Notes: The top panel plots the neutrality curves in the integrated framework. The bottom left panel plots entrant profit with and without the merger under an assumption that the merger does not create efficiencies. The bottom right panel plots the percentage change in entrant profit due to a merger without efficiencies, along with the analogous percentage change due to a merger with lower bound efficiencies. The figure is generated numerically given subscriber shares for the mobile wireless telecommunications industry.

we assume a DISH type of $T^F = 5.00$. Applying Proposition 1, this exceeds the type of the compensating entrant because the T-Mobile type is $T^2 = 6.14$.

We first simulate the T-Mobile/Sprint merger under the assumption of merger-induced entry by DISH. The results indicate that the prices of T-Mobile and Sprint increase by 3.1%

and 4.4%, respectively. Consumer surplus nonetheless increases by 5.0% due to the diversity that DISH introduces. Whether merger-induced entry would occur is another matter. Indeed, we find no SPE featuring merger-induced entry because the DISH type is too high, as we have $T^F = 5.00 > \hat{T}^F = 4.63$. With a small efficiency, say a 2% reduction in marginal costs, the merger would induce entry by DISH (with small enough barriers), but DISH's entry would make the merger unprofitable. With a large efficiency, say a 4% reduction in marginal costs, the merger would be profitable even with merger-induced entry by DISH, but it also reduces the profitability of DISH entry. In neither scenario does both merger and merger-induced entry occur. Thus, our analysis points to efficiencies, rather than merger-induced entry by DISH, as the more important consideration for the T-Mobile/Sprint merger.[31]

Our analysis thus far has not incorporated the divestiture of the Boost brand from Sprint to DISH, which was intended to help facilitate merger-induced entry. To account for the divestiture, we explore a range of possibilities in which we transfer some of the merged firm's type to DISH. Specifically, we look at transfers that range between 0% and 50% of the Sprint type. For each transfer, we also consider efficiencies between 0% and 100% of the compensating efficiency. Across these combinations of divestitures and efficiencies, we find no SPE featuring merger-induced entry in which pre-merger consumer surplus is preserved. We conclude that making some allowance for the divestiture does not change our result that DISH was unlikely to be a merger-induced entrant.

To the extent that evidence pointed toward DISH entry, our analysis indicates that entry may have occurred even without the T-Mobile/Sprint merger. For such cases, the 2023 US Merger Guidelines suggest analyzing the effects of the merger taking into account DISH as a competitor both before and after the merger. Thus, we proceed by treating DISH as an incumbent.[32] When we do so, our simulation results indicate that, absent efficiencies, the merger increases the prices of T-Mobile and Sprint by 4.0% and 5.2%, respectively, and decreases consumer surplus by 1.7%. The compensating efficiency needed to offset consumer surplus loss is reduced relative to an equilibrium that does not feature DISH as a competitor.

5   Conclusion

We provide a useful framework for the analysis merger effects accounting for entry and efficiencies. We demonstrate how one can use our framework to inform (1) how much entry is needed to preserve consumer surplus, (2) how to analyze efficiencies and entry jointly, and (3) how to bound the entry costs under which merger-induced entry occurs. Using our

---

[31]Asker and Katz (2023) summarize some of the *ex ante* evidence about efficiencies.

[32]The Merger Guidelines, §4.4.A, are explicit on this point, stating that "[f]irms not currently supply products in the relevant market, but that have committed to entering the market in the near future, are also considered market participants."

model, we characterize the conditions under which mergers induce entry that is sufficient to preserve consumer surplus, given Bertrand price competition and an array of different demand systems. We find that, absent merger efficiencies or fixed cost savings, for such an outcome to obtain in SPE requires that the entrant be a distant competitor to the merging firms. We also show that mergers generate weaker incentives for entry, the more distant is the prospective entrant. This creates the tension that merger-induced entry sufficient to preserve consumer surplus requires that the entrant be a distant competitor of the merging firms, yet mergers may do little to induce entry by such a firm.

The generality of the model has some limitations that create opportunities for further research. To our knowledge, the academic literature has not examined empirically the magnitude of fixed cost savings that are created by mergers. Some entry costs could be endogenous due to strategic actions by incumbents to deter or punish entry; these are not incorporated into our model. Furthermore, our model does not address uncertainty or private information about the likelihood of entry, about the strength of the entrant, or about the magnitude of efficiencies or fixed cost savings. Other modeling frameworks also could be examined, including auction models in which buyers set reserve prices to discipline the post-merger market power of sellers (e.g., Waehrer and Perry (2003); Loertscher and Marx (2019)). We view these as providing interesting possibilities for future research.

References

Aliprantis, C. D. and K. C. Border (2006). *Infinite Dimensional Analysis: A Hitchhiker's Guide*. Springer Science & Business Media.

An, Y. and W. Zhao (2019). Dynamic efficiencies of the 1997 boeing-mcdonnell douglas merger. *RAND Journal of Economics 50*(3), 666–694.

Anderson, S. P., N. Erkal, and D. Piccinin (2020). Aggregative games and oligopoly theory: Short-run and long-run analysis. *RAND Journal of Economics 51*(2), 470–495.

Ashenfelter, O., D. Hosken, and M. Weinberg (2015). Efficiencies brewed: Pricing and consolidation in U.S. brewing. *RAND Journal of Economics 46*(2), 328–361.

Asker, J. and M. L. Katz (2023). Efficiencies, remedies, and competition: The Sprint/T-Mobile merger. *Antitrust Economics at a Time of Upheaval: Recent Competition Policy Cases on Two Continents*, John E. Kwoka, Tommaso M. Valletti, and Lawrence J. White (eds). Competition Policy International.

Berry, S., J. Levinsohn, and A. Pakes (1995). Automobile prices in market equilibrium. *Econometrica 63*(4), 847–890.

Berry, S. T. (1994). Estimating Discrete-Choice Models of Product Differentiation. *RAND Journal of Economics 25*(2), 242–262.

Braguinsky, S., A. Ohyama, T. Okazaki, and C. Syverson (2015). Acquisitions, productivity, and profitability: Evidence from the Japanese cotton spinning industry. *American Economic Review 105*(7), 2086–2119.

Bronnenberg, B. J., M. W. Kruger, and C. F. Mela (2008). The IRI marketing data set. *Marketing Science 27*(4), 745–748.

Cabral, L. (2003). Horizontal mergers with free-entry: Why cost efficiencies may be a weak defense and asset sales a poor remedy. *International Journal of Industrial Organization 21*, 607–623.

Cardell, S. N. (1997). Variance Components Structures for the Extreme-Value and Logistic Distributions with Applications to Models of Heterogeneity. *Econometric Theory 13*, 185–213.

Ciliberto, F., C. Murry, and E. Tamer (2021). Market structure and competition in airline markets. *Journal of Political Economy 129*(11), 2995–3038.

Conlon, C. T. and J. H. Mortimer (2021). Empirical properties of diversion ratios. *RAND Journal of Economics 52*(4), 693–726.

Davidson, C. and A. Mukherjee (2007). Horizontal mergers with free entry. *International Journal of Industrial Organization 25*, 157–172.

Dermirer, M. and O. Karaduman (2022). Do mergers and acquisitions improve efficiency: Evidence from power plants. Working Paper.

Erkal, N. and D. Piccinin (2010). Welfare-reducing mergers in differentiated oligopolies with free entry. *Economic Record 86*(273), 178–184.

Fan, Y. and C. Yang (2023). Estimating discrete games with many firms and many decisions: An application to merger and product variety. Working Paper.

Farrell, J. and C. Shapiro (1990). Horizontal mergers: An equilibrium analysis. *American Economic Review 80*(1), 107–126.

Hirsch, M. W. (2012). *Differential topology*, Volume 33. Springer Science & Business Media.

Kulick, R. B. (2017). Ready-to-mix: Horizontal mergers, prices, and productivity. US Census Bureau Center for Economic Studies Paper No. CES-WP-17-38.

Li, S., J. Mazur, J. Roberts, Y. Park, A. Sweeting, and J. Zhang (2022). Repositioning and market power after airline mergers. *RAND Journal of Economics 53*(1), 166–199.

Loertscher, S. and L. M. Marx (2019). Merger review for markets with buyer power. *Journal of Political Economy 127*(6), 2967–3017.

Mas-Colell, A. (1974). A note on a theorem of F. Browder. *Mathematical Programming 6*(1), 229–233.

Miller, N. H. and G. Sheu (2021). Quantitative methods for evaluating the unilateral effects of mergers. *Review of Industrial Organization 58*(1), 143–177.

Miller, N. H. and M. C. Weinberg (2017). Understanding the price effects of the MillerCoors joint venture. *Econometrica 85*(6), 1763–1791.

Nevo, A. (2001). Measuring market power in the ready-to-eat cereal industry. *Econometrica 69*(2)*, pp. 307–342.

Nevo, A. and F. Rossi (2008). An approach for extending dynamic models to settings with multi-product firms. *Economics Letters 100*, 49–52.

Nocke, V. and N. Schutz (2018). Multiproduct-firm oligopoly: An aggregative games approach. *Econometrica 86*(2), 523–557.

Nocke, V. and N. Schutz (2019). An aggregative games approach to merger analysis in multiproduct-firm oligopoly. Working Paper.

Nocke, V. and M. Whinston (2022). Concentration screens for horizontal mergers. *American Economic Review 112*(6), 1915–1948.

Shapiro, C. (2010). The 2010 Horizontal Merger Guidelines: From hedgehog to fox in forty years. *Antitrust Law Journal 77*, 49–107.

Spector, D. (2003). Horizontal mergers, entry, and efficiency defences. *International Journal of Industrial Organization 21*, 1591–1600.

Starc, A. and T. G. Wollman (2023). Does entry remedy collusion? Evidence from the generic prescription drug cartel. Working Paper.

Stigler, G. J. (1950). Monopoly and oligopoly by merger. *American Economic Review (Papers & Proceedings) 40*(2), 23–34.

Waehrer, K. and M. K. Perry (2003). The effects of mergers in open-auction markets. *RAND Journal of Economics 34*(2), 287–304.

Werden, G. J. (1996). A robust test for consumer welfare enhancing mergers among sellers of differentiated products. *The Journal of Industrial Economics 44*(4), pp. 409–413.

Werden, G. J. and L. M. Froeb (1994). The effects of mergers in differentiated products industries: Logit demand and merger policy. *Journal of Law, Economics, and Organization 10*(407), 407–426.

Werden, G. J. and L. M. Froeb (1998). The entry-inducing effects of horizontal mergers: An exploratory analysis. *The Journal of Industrial Economics 46*(4), pp. 525–543.

Werden, G. J. and L. M. Froeb (2002). The antitrust logit model for predicting unilaterial competitive effects. *Antitrust Law Journal 70*(1), 257–260.

Appendix

For Online Publication

A Notes on Aggregative Games

In this appendix, we derive the aggregative games formulation of the Bertrand model with MNL, CES, NMNL, and NCES demand. We focus especially on the MNL and CES models in order to provide something of a "practitioner's guide" for those who previously have not studied aggregative games.

A.1 MNL Demand

We take as given the profit function and first order conditions of (1) and (2), the indirect utility of (3), and the market shares of (4). In this framework, it is well known that consumer surplus is given by

$$CS = \frac{1}{\alpha} \ln \left(1 + \sum_{j \in \mathcal{J}} \exp(v_j - \alpha p_j) \right). \tag{A.1}$$

The primitives of the aggregative game reformulation are the vector of firm-specific types, $\{T^f\} \ \forall f \in \mathcal{F}$, and the price parameter, α. Equation (6) defines the type of each firm f as

$$T^f \equiv \sum_{j \in \mathcal{J}^f} \exp(v_j - \alpha c_j),$$

which represents the firm's contribution to consumer surplus if its prices equal its marginal costs. From these primitives, the Bertrand equilibrium can be characterized as a vector of "ι-markups," $\{\mu^f\} \ \forall f \in \mathcal{F}$, a vector of firm-level market shares, $\{s^f\} \ \forall f \in \mathcal{F}$, and a market aggregator, H. We define markups below, and let $s^f = \sum_{j \in \mathcal{J}^f} s_j$. The aggregator is defined as $H \equiv 1 + \sum_{j \in \mathcal{J}} \exp(v_j - \alpha p_j)$, which is the denominator from the market share formula of the product-level model (see (4)).

We first derive a relationship between the ι-markups and firm-level market shares. The product-specific price derivatives for logit demand are

$$\frac{\partial s_j}{\partial p_k} = \begin{cases} -\alpha s_j (1 - s_j) & \text{if } k = j \\ \alpha s_j s_k & \text{if } k \neq j. \end{cases}$$

Substituting these demand derivatives into the first order conditions of (2) for some product j and rearranging gives

$$\alpha(p_j - c_j) = 1 + \alpha \sum_{k \in \mathcal{J}^f} (p_k - c_k) s_k. \tag{A.2}$$

The right-hand side of this equation does not depend on the which product $j \in \mathcal{J}^f$ is referenced. This implies that the left-hand side is equivalent for all products sold by firm f,

meaning each firm imposes a common markup (in levels) across all of its products. Following (7), define the ι-markup of firm f as $\mu^f \equiv \alpha(p_j - c_j) \ \forall j \in \mathcal{J}^f$. Substituting back into (A.2) obtains an equilibrium relationship between markups and shares:

$$\mu^f = \frac{1}{1 - s^f}. \tag{A.3}$$

We also have $s^f = (1/H) \sum_{j \in \mathcal{J}^f} \exp(v_j - \alpha p_j)$ from (4), after substituting in for the definition of the aggregator, H. Adding and subtracting αc_j inside the exponential and applying the definitions of μ^f and T^f gives

$$s^f = \frac{T^f}{H} \exp(-\mu_f) \tag{A.4}$$

$$\Longleftrightarrow \quad \frac{T^f}{H} = s^f \exp\left(\frac{1}{1 - s^f}\right). \tag{A.5}$$

Plugging (A.4) into (A.3), we obtain that equilibrium ι-markups satisfy (9):

$$\mu_f \left(1 - \frac{T^f}{H} \exp(-\mu_f)\right) = 1.$$

Let the unique solution for μ_f from this expression be written as $m(T^f/H)$. This *markup fitting-in function*, $m(\cdot)$, has the properties that $m(0) = 1$ and $m'(\cdot) > 0$. Plugging $\mu_f = m(T^f/H)$ into (A.4) yields the expression for equilibrium market shares provided in (10). Equilibrium market shares can be written $s^f = S(T^f/H)$, and thus equilibrium profit can be written $\Pi^f = \pi(T^f/H)$. To close the system, the aggregator satisfies an adding-up constraint of (11). The expressions for equilibrium profit and consumer surplus provided in (12) obtain immediately.

A.2 CES

Derivation of the CES aggregative game mirrors that of MNL case, except the CES demand derivatives and formula for shares must be used instead. With CES, the pricing first order condition for product j becomes

$$\sigma \frac{p_j - c_j}{p_j} = 1 + (\sigma - 1) \sum_{k \in \mathcal{J}^f} s_k \frac{p_k - c_k}{p_k}, \tag{A.6}$$

which is the counterpart to the MNL equation (A.2). We again see that the right-hand side of this equation does not depend on the identity of $j \in \mathcal{J}^f$, which in turn implies that each firm charges a constant percentage markup across all of its products.

Once we define the ι-markup as $\mu^f = \sigma(p_j - c_j)/p_j$ following (7), we obtain

$$\mu^f = \frac{1}{1 - \frac{\sigma - 1}{\sigma} s^f} \tag{A.7}$$

after substituting into the pricing first order condition. Take the share equation (5) and

36

multiply and divide it by $c_j^{1-\sigma}$. We can then substitute in the definitions of the aggregator H, μ^f, and the type T^f. Summing across the shares for the products sold by firm f gives

$$s^f = \frac{T^f}{H}\left(1 - \frac{\mu^f}{\sigma}\right)^{\sigma-1} \tag{A.8}$$

for firm-level revenue shares. Substituting this share into the markup expression in (A.7) gives the markup fitting-in function,

$$1 = \mu^f\left(1 - \frac{\sigma-1}{\sigma}\frac{T^f}{H}\left(1 - \frac{\mu^f}{\sigma}\right)^{\sigma-1}\right) \tag{A.9}$$

which appears in (9). The model is closed with the adding-up constraint given by (11).

A.3 NMNL

With NMNL demand, the following equations hold in Bertrand equilibrium:

$$\mu^f = \frac{1}{1 - \rho s^{f|g} - (1-\rho)s^f} \tag{A.10}$$

$$1 = \mu^f\left(1 - \rho\frac{T^f}{H_g}\exp(-\mu^f) - (1-\rho)\frac{T^f}{H_g}\frac{H_g^{1-\rho}}{H}\exp(-\mu^f)\right) \tag{A.11}$$

$$\frac{T^f}{H_g} = s^{f|g}\exp\left(\frac{1}{1 - \rho s^{f|g} - (1-\rho)s^f}\right) \tag{A.12}$$

$$\bar{s}_g = \frac{H_g^{1-\rho}}{H} \tag{A.13}$$

$$s^f = s^{f|g}\bar{s}_g \tag{A.14}$$

$$1 = \sum_{f\in\mathcal{F}_g} s^{f|g} \tag{A.15}$$

$$\frac{1}{H} = 1 - \sum_{f\in\mathcal{F}} s^f \tag{A.16}$$

$$\pi^f = \frac{1-\rho}{\alpha}\mu^f s^f \tag{A.17}$$

$$CS = \frac{1}{\alpha}\ln(H) \tag{A.18}$$

where T^f is the type of the firm, s^f is the share of the firm, $s^{f|g}$ is the share of the firm within its nest, $\bar{s}_g$ is the share of the nest, μ^f is the ι-markup of the firm, H_g is a nest aggregator, H is the market aggregator, π^f is the profit of the firm, and CS is consumer surplus.

Firm types are defined as in (23). Firm share is given by $s^f = \sum_{j\in\mathcal{J}^f} s_j$, as in the MNL and CES models. Firm share within its nest is given by $s^{f|g} = \sum_{j\in\mathcal{J}^f} s_{j|g}$, where the share of

a product within a nest is

$$s_{j|g} = \frac{\exp\left(\frac{v_j - \alpha p_j}{1-\rho}\right)}{H_g}. \tag{A.19}$$

The aggregators are defined as $H_g \equiv \sum_{j \in \mathcal{J}_g} \exp((v_j - \alpha p_j)/(1-\rho))$ and $H \equiv 1 + \sum_{g \in \mathcal{G}} H_g^{1-\rho}$. The markup is defined as $\mu^f \equiv (\alpha/(1-\rho))(p_j - c_j)$ for all $j \in \mathcal{J}^f$.

The pricing first order condition for good j can be written as

$$\frac{\alpha}{1-\rho}(p_j - c_j) = 1 + \frac{\alpha\rho}{1-\rho}\sum_{k \in \mathcal{J}^f}(p_k - c_k)s_{k|g} + \alpha\sum_{k \in \mathcal{J}^f}(p_k - c_k)s_k, \tag{A.20}$$

under the assumption that firm f owns products only in nest g. We again see that the right-hand side of this condition does not depend on the identity of $j \in \mathcal{J}^f$. Substituting in for the definition of μ^f gives (A.10).

Next, adding and subtracting αc_j inside the exponential on the right-hand side of (A.19) and applying the definitions of μ^f, T^F, and H_g obtains

$$s^{f|g} = \frac{T^f}{H_g}\exp(-\mu^f), \tag{A.21}$$

which rearranges to (A.12). Then (A.11) can be obtained by plugging (A.12) and (A.13) back into (A.10). Next, (A.15) and (A.16) are adding-up constraints that close the model, (A.17) is obtained by plugging μ^f into the profit function, and (A.13), (A.14), and (A.18) follow directly from the NMNL functional form.

A.4 NCES

With NCES, the following equations hold in Bertrand equilibrium:

$$\mu^f = \frac{1}{1 - \frac{\gamma-1}{\sigma}s^f - \frac{\sigma-\gamma}{\sigma}s^{f|g}} \tag{A.22}$$

$$1 = \mu^f \left(1 - \frac{\gamma-1}{\sigma}\frac{T^f}{H_g^{\frac{\sigma-\gamma}{\sigma-1}}H}\left(1 - \frac{\mu^f}{\sigma}\right)^{\sigma-1} - \frac{\sigma-\gamma}{\sigma}\frac{T^f}{H_g}\left(1 - \frac{\mu^f}{\sigma}\right)^{\sigma-1}\right) \tag{A.23}$$

$$\frac{T^f}{H_g} = s^{f|g}\left(1 - \frac{\mu^f}{\sigma}\right)^{1-\sigma} \tag{A.24}$$

$$s^f = \frac{T^f}{H_g^{\frac{\sigma-\gamma}{\sigma-1}}H}\left(1 - \frac{\mu^f}{\sigma}\right)^{\sigma-1} \tag{A.25}$$

$$1 = \sum_{f\in\mathcal{F}_g} s^{f|g} \tag{A.26}$$

$$\frac{1}{H} = 1 - \sum_{f\in\mathcal{F}} s^f \tag{A.27}$$

$$\pi^f = \frac{1}{\sigma}\mu^f s^f \tag{A.28}$$

$$CS = H^{1/(\gamma-1)} \tag{A.29}$$

where T^f is the type of the firm, s^f is the share of the firm, $s^{f|g}$ is the share of the firm within its nest, μ^f is the ι-markup of the firm, H_g is a nest aggregator, H is the market aggregator, π^f is the profit of the firm, and CS is consumer surplus.

Firm types are defined as in (23). Firm share is given by $s^f = \sum_{j\in\mathcal{J}^f} s_j$, as in the MNL and CES models. Firm share within its nest is given by $s^{f|g} = \sum_{j\in\mathcal{J}^f} s_{j|g}$, where the share of a product within a nest is

$$s_{j|g} = \frac{v_j p_j^{1-\sigma}}{\sum_{k\in\mathcal{J}_g} v_k p_k^{1-\sigma}} \tag{A.30}$$

The aggregators are defined as $H_g \equiv \sum_{j\in\mathcal{J}_g} v_j p_j^{1-\sigma}$ and $H \equiv 1 + \sum_{g\in\mathcal{G}} H_g^{\gamma-1/\sigma-1}$. The markup is defined as $\mu^f \equiv \sigma(p_j - c_j)/p_j$ for all $j \in \mathcal{J}^f$, same as with CES demand.

The pricing first order condition for good j can be written as

$$\sigma\frac{p_j - c_j}{p_j} = 1 + \sum_{k\in\mathcal{J}^f}\frac{p_k - c_k}{p_k}[(\gamma-1)s_k + (\sigma-\gamma)s_{k|g}] \tag{A.31}$$

under the assumption that firm f owns products only in nest g. We again see that the right-hand side of this condition does not depend on the identity of $j \in \mathcal{J}^f$. Substituting in for the definition of μ^f gives (A.22).

Next, multiplying and dividing by $c_j^{1-\sigma}$ on the right-hand side of (A.30) and applying the

definitions of μ^f, T^F, and H_g obtains

$$s^{f|g} = \frac{T^f}{H_g} \left(1 - \frac{\mu^f}{\sigma} \right)^{\sigma-1}. \tag{A.32}$$

which rearranges to (A.24). Applying the same computation to (21) gives (A.25). Then (A.23) can be obtained by plugging (A.24) and (A.25) back into (A.22). Next, (A.26) and (A.27) are adding-up constraints that close the model, (A.28) is obtained by plugging μ^f into the profit function, and (A.29) follows directly from the NCES functional form.

B Delayed and Probabilistic Entry

Our baseline model considers a three-stage game in which (1) firms decide to merge, (2) an outside decides to enter, and (3) payoffs are realized according to a differentiated pricing game. In this appendix, we consider two variants. The first is a model of *delayed entry* in which incumbents obtain payoffs for N periods before entry occurs (if it does occur). The second is a model of *probabilistic entry* in which entry occurs in the second stage with some fixed probability p if it is profitable, and with probability zero otherwise.

With delayed and probabilistic entry, a merger that induces entry increases the net present value of the merging firms if and only if

$$\frac{1-\theta}{1-\delta}\pi^M_{m,ne} + \frac{\theta}{1-\delta}\pi^M_{m,e} \geq \sum_{i=1,2} \frac{1}{1-\delta}\pi^i_{nm,ne} \tag{B.1}$$

where δ is a discount factor, $\theta = \delta^N$ with delayed entry, and $\theta = p$ with probabilistic entry. Similarly, a merger that induces entry increases the net present value of consumer surplus if and only if

$$\frac{1-\theta}{1-\delta}CS^{m,ne} + \frac{\theta}{1-\delta}CS^{m,e} \geq \frac{1}{1-\delta}CS^{nm,ne} \tag{B.2}$$

As these equations nest both delayed and probabilistic entry, we proceed by analyzing mergers and entry in the two models jointly.

With $\theta = 1$, the analytical results from in the main body of the paper obtain, and with MNL or CES demands merger-induced entry sufficient to preserve consumer surplus renders merger unprofitable. At the other end, entry is irrelevant with $\theta = 0$.

With $\theta \in (0, 1)$, our intuition is that Proposition 1 extends for most reasonable parameterizations. The reason is that as θ decreases from one, the profitability of the merger increases but so does the consumer surplus loss. Given the strict inequalities we obtain, the first of these effects would have to be considerably stronger than the second to generate a profitable, procompetitive merger. Our examination of the implied relationships indicates this is unlikely to be the case.

In support of this conjecture, we conduct numerical simulations using a model with two incumbents and MNL demand. We consider market shares for the incumbents that range from 0.01 to 0.80. After calibrating incumbent types, we examine entrants with types that range between that of the compensating entrant (Proposition 1) and ten times that of the merged

firm. Finally, for each of these, we scale θ between zero and one in increments of 0.01. We find no cases in which a profitable merger increases consumer surplus.

This is not to claim that profitable, pro-competitive mergers cannot be found with *unreasonable* parameterizations. Indeed, for any initial set of incumbent types and MNL or CES demands, we can prove that there exists some θ and entrant type T^F for which a profitable merger improves consumer surplus. As one example, suppose that two incumbents each have of a market share of 0.40 initially. The implied types are $T^1 = T^2 = 10.59$. Further let $\theta = 0.099$, which obtains with 21.96 years of delay (given $\delta = 0.90$) or with a probability of post-merger entry just less than 10 percent. If, in addition, the entrant's type exceeds 3.59×10^{102}, then a profitable, pro-competitive merger obtains.[33] This entrant captures a market share of 0.996; the incumbents' combined market share decreases to 0.004 and the share of the outside good is approximately zero.

We now formally state that with delayed and probabilistic entry, the model can generate profitable, pro-competitive mergers.

Proposition B.1. *Fix an initial market structure comprising $f = 1, \ldots, F - 1$ incumbents and their types, and consider a merger of firms 1 and 2. With MNL and CES demands, there exists a θ and entrant type T^F such that merger with induced entry increases the present value of consumer surplus and the merging firms' profit.*

Proof. See Appendix C. □

For intuition, if θ is small enough—i.e., entry is sufficiently delayed or unlikely—then a merger increases the present value of the merging firms' profit, even if this profit is approximately zero in every period after entry occurs. Thus for any baseline calibration, by choosing a small enough θ, the profit and surplus inequalities 'decouple,' in the sense that the profit inequality holds for *any* value of entrant type. However, as consumer surplus increases to infinity with the type of the entrant, one can then always find some sufficiently capable entrant such that the present value of consumer surplus increases. The numerical results we describe above suggest that this theoretical possibility is not practically relevant for merger review.

C Section 3 Proofs

C.1 Lemmas

Lemma C.1. *Let firm f be a non-merging incumbent with $T^f > 0$, and let $*$ denote either 'merger, no entry' or 'merger, entry.' Then if any of the following conditions holds, all of the following conditions hold for that case.*

(i) The merger does not affect the profitability of firm f.

(ii) The merger does not affect consumer surplus.

(iii) Market shares satisfy the following equality:

$$s^1_{nm,ne} + s^2_{nm,ne} = s^M_* + \sum_{\{f \in \mathcal{F}_* \backslash \mathcal{F}_{nm,ne} | f \neq m\}} s^f_*.$$

---

[33]For comparison, there are approximately 2.40×10^{67} atoms in the Milky Way galaxy.

Proof. **(i)** $\implies$ **(ii)**: Suppose (i) holds, that is:

$$\pi^f_{nm,ne} = \pi^f_*$$

By (12), $\mu^f_{nm,ne} = \mu^f_*$, and by (A.3), $s^f_{nm,ne} = s^f_*$. Because $T^f = T^f_{nm,ne} = T^f_*$ by hypothesis, (10) implies:

$$\frac{T^f}{H_{nm,ne}} = s^f_{nm,ne} \exp\left(\frac{1}{1 - s^f_{nm,ne}}\right) = s^f_* \exp\left(\frac{1}{1 - s^f_*}\right) = \frac{T^f}{H_*}$$

and thus $H_{nm} = H_m$, which implies (ii).

(ii) $\implies$ **(i)**: Suppose now that $H_{nm,ne} = H_* = H$. By (10), we obtain $s^f_{nm,ne} = s^f_*$ for every $f \in \mathcal{F}_{nm,ne}$ immediately, and (i) follows by a chain of substitutions identical to the above.

(ii) $\implies$ **(iii)**: Suppose now that $H_{nm,ne} = H_* = H$. From (11):

$$\frac{1}{H} + \sum_{f\in\mathcal{F}_{nm,ne}} s^f_{nm,ne} = \frac{1}{H} + \sum_{f\in\mathcal{F}_*} s^f_* \iff \sum_{f\in\mathcal{F}_{nm,ne}} s^f_{nm,ne} = \sum_{f\in\mathcal{F}_*} s^*_m,$$

which implies (iii) immediately upon cancelling terms (via appeal to (ii) implying (i) and hence to the shares also coinciding across scenarios).

(iii) $\implies$ **(ii)**: We proceed by contraposition. Thus suppose that the merger affects consumer surplus: $H_{nm,ne} \neq H_*$. Let f belong to both $\mathcal{F}_{nm,ne}$ and $\mathcal{F}_*$, i.e. let f denote any firm other than $1, 2, M$ or potentially F. By (10), we have:

$$\frac{T^f}{H_{nm,ne}} = s^f_{nm,ne} \exp\left(\frac{1}{1 - s^f_{nm,ne}}\right)$$

and

$$\frac{T^f}{H_*} = s^f_* \exp\left(\frac{1}{1 - s^f_*}\right).$$

For both equations, the right-hand side is strictly increasing in the relevant share, and thus for all such f:

$$\frac{1}{H_{nm,ne}} > \frac{1}{H_*} \iff s^f_{nm,ne} > s^f_*.$$

Thus:

$$\frac{1}{H_{nm,ne}} + \sum_{f\in\mathcal{F}_{nm,ne}\cap\mathcal{F}_*} s^f_{nm,ne} \neq \frac{1}{H_*} + \sum_{f\in\mathcal{F}_{nm}\cap\mathcal{F}_*} s^f_*,$$

and it follows by (11) that (iii) cannot hold. $\square$

Lemma C.2. *In Bertrand equilibrium with MNL demand, all firms with positive share have markups such that $\mu^f \in (1, \infty)$. If we instead have CES demand, all firms with positive share have markups such that $\mu^f \in (1, \sigma)$.*

Proof. In equilibrium in the MNL case we have that

$$\mu^f = \frac{1}{1 - s^f}$$

from (A.3). There is an outside good with positive share, so $s^f < 1$ for all active firms. Thus we have that $\mu^f > 1$, since the denominator in the expression above, $1 - s^f$, is less than one for all positive values of s^f. We also have that μ^f approaches infinity as s^f approaches 1.

In equilibrium in the CES case we have that

$$\mu^f = \frac{1}{1 - \frac{\sigma-1}{\sigma} s^f} = \frac{\sigma}{\sigma - s^f(\sigma - 1)}$$

from (A.7). Given that there is an outside good with positive share, $s^f < 1$ for all active firms. Thus, the first equality implies that $\mu^f > 1$, since the denominator $1 - ((\sigma - 1)/\sigma)s^f$ is less than one for all positive values of s^f. The second equality implies that μ^f is bounded above by σ as s^f approaches 1. $\qquad\square$

Lemma C.3. *Define the function*

$$\phi(x) \equiv \begin{cases} xe^{-x} & \text{(MNL or NMNL)} \\ x\left(1 - \frac{x}{\sigma}\right)^{\sigma-1} & \text{(CES or NCES)} \end{cases} \tag{C.1}$$

where the first specification applies to the MNL and NMNL models, and the second applies to the CES and NCES models. This function $\phi(\cdot)$ is decreasing on $(1, \infty)$ for the MNL/NMNL specification and decreasing on $(1, \sigma)$ for the CES/NCES specification.

Proof. The derivative for the MNL/NMNL specification is

$$\frac{d}{dx}\phi(x) = (1 - x)\exp(-x).$$

This derivative is negative if and only if $1 - x$ is negative. This in turn is true if $x > 1$.

For the CES/NCES specification, we employ a change of variables by defining $\tilde{x} = x/\sigma$. The derivative of the redefined function has the same sign as the original, since σ is positive. We have that $\phi(\tilde{x}) = \sigma\tilde{x}(1 - \tilde{x})^{\sigma-1}$. Then the derivative is

$$\frac{d}{d\tilde{x}}\phi(\tilde{x}) = \sigma(1 - \tilde{x})^{\sigma-1}\left[1 - \frac{\tilde{x}(\sigma - 1)}{1 - \tilde{x}}\right].$$

This derivative is negative in the relevant range if and only if the term in brackets is negative, because $(1 - \tilde{x})$ is positive for all $x \in (1, \sigma)$. The term in brackets is negative if and only if $\tilde{x} > 1/\sigma$. We know that $x > 1$, so this condition is met. $\qquad\square$

C.2 Proof of Proposition 1

Proposition 1: With MNL demand, the type of the compensating entrant, $\widetilde{T}^F$, satisfies

$$\frac{\widetilde{T}^F}{T^1 + T^2} = \frac{(s^1 + s^2 - s^M)\exp\left(\frac{1}{1-s^1-s^2+s^M}\right)}{s^1 \exp\left(\frac{1}{1-s^1}\right) + s^2 \exp\left(\frac{1}{1-s^2}\right)} \tag{C.2}$$

where s^M is the unique solution to

$$s^M \exp\left(\frac{1}{1-s^M}\right) = s^1 \exp\left(\frac{1}{1-s^1}\right) + s^2 \exp\left(\frac{1}{1-s^2}\right). \tag{C.3}$$

With CES demand, the type of the compensating entrant satisfies

$$\frac{\widetilde{T}^F}{T^1 + T^2} = \frac{(s^1 + s^2 - s^M)\left(\sigma + \frac{s^1+s^2-s^M}{1-(s^1+s^2-s^M)}\right)^{\sigma-1}}{s^1\left(\sigma + \frac{s^1}{1-s^1}\right)^{\sigma-1} + s^2\left(\sigma + \frac{s^2}{1-s^2}\right)^{\sigma-1}} \tag{C.4}$$

where s^M is the unique solution to

$$s^M\left(\sigma + \frac{s^M}{1-s^M}\right)^{\sigma-1} = s^1\left(\sigma + \frac{s^1}{1-s^1}\right)^{\sigma-1} + s^2\left(\sigma + \frac{s^2}{1-s^2}\right)^{\sigma-1}. \tag{C.5}$$

Furthermore, with MNL and CES demand, $\widetilde{T}^F < \frac{1}{2}\left(T^1 + T^2\right)$ and $s^F < \frac{1}{2}\left(s^1 + s^2\right)$.

Proof. Begin by rearranging (10) to solve for firm type, giving

$$T^f = \begin{cases} Hs^f \exp\left(\frac{1}{1-s^f}\right) & \text{(MNL)} \\ Hs^f(\sigma-1)^{1-\sigma}\left(\sigma + \frac{s^f}{1-s^f}\right)^{\sigma-1} & \text{(CES)} \end{cases} \tag{C.6}$$

after substituting in for markups. Then evaluate this type equation for firm F after the merger and firms 1 and 2 before the merger, substituting in for the entrant share using $s^F = s^1 + s^2 - s^M$, which obtains from Lemma C.1. Dividing the result for firm F by the sum of the results for firms 1 and 2 gives (C.2) and (C.4) for MNL and CES, respectively.

Without efficiencies, $T^M = T^1 + T^2$. Substituting into this sum for types using (C.6) gives (C.3) and (C.5). These two expressions have unique positive solutions because the expressions $x\exp(1/(1-x))$ and $x(\sigma + x/(1-x))^{\sigma-1}$ are increasing if $x \in [0,1)$.

To establish the final claim, let s^a be the average of s^1 and s^2, calculated as $(s^1 + s^2)/2$. Let T^a be the type that generates a share of s^a given aggregator H, which can be found by solving (10) holding H fixed. (Note that if $s^1 \geq s^2$, then $s^1 \geq s^a \geq s^2$ and $T^1 \geq T^a \geq T^2$, the latter due to the monotonicity of shares in terms of T^f/H.) We wish to characterize the relationship between the entrant's type T^F and this "average" type T^a. In order for consumers to be unharmed, H must be unchanged due to the merger. Therefore, since $T^M > T^1$ and $T^M > T^2$, $T^M/H > T^1/H$ and $T^M/H > T^2/H$. In turn, this means that $s^M > s^1$ and $s^M > s^2$, since shares are increasing in T^f/H. Adding these inequalities gives $2s^M > s^1 + s^2$,

44

and then dividing by two gives $s^M > s^a$. As shown by Lemma C.1, if H remains the same, then $s^F + s^M = s^1 + s^2$, which also means that $s^F + s^M = 2s^a$. In order for this equality to hold when we also know that $s^M > s^a$, it must be that $s^F < s^a$. By the monotonicity of shares, this means that $T^F < T^a$. $\qquad\square$

C.3 Proof of Proposition 2

Proof. Suppose, for purposes of contradiction, there exists a SPE in which firms 1 and 2 merge, and consumers surplus does not decrease. Thus the merger must increase joint profits:

$$\pi^M \left(\frac{T^1 + T^2}{H_{m,e}} \right) \geq \pi^1 \left(\frac{T^1}{H_{nm,ne}} \right) + \pi^2 \left(\frac{T^2}{H_{nm,ne}} \right),$$

where $H_{nm,ne}$ denotes the aggregator with no merger and no entry. By hypothesis, consumers surplus does not fall, hence we have $H_{nm,e} \leq H_{m,e}$. Furthermore, by Nocke and Schutz (2018, Proposition 6), π^M is decreasing in H all else equal, meaning that

$$\pi^M \left(\frac{T^1 + T2}{H_{nm,ne}} \right) \geq \pi^1 \left(\frac{T^1}{H_{nm,ne}} \right) + \pi^2 \left(\frac{T^2}{H_{nm,ne}} \right). \tag{C.7}$$

Multiplying the markup and firm share shows that firm profit is given by

$$\pi^f = \begin{cases} \frac{1}{\alpha} \mu^f \frac{T^f}{H} \exp(-\mu^f) & \text{(MNL)} \\ \frac{1}{\sigma} \mu^f \frac{T^f}{H} \left(1 - \frac{\mu^f}{\sigma} \right)^{\sigma-1} & \text{(CES)}. \end{cases}$$

Then (C.7) is satisfied, after canceling certain constants, if and only if

$$(T^1 + T^2)\phi\big(m(T^1 + T^2, H^{nm,ne})\big) \geq T^1 \phi\big(m(T^1, H^{nm,ne})\big) + T^2 \phi\big(m(T^2, H^{nm,ne})\big),$$

where $\phi(\cdot)$ is defined as in C.1, and $m(\cdot)$ denotes the markup fitting-in function for the MNL or CES, as appropriate. This expression is equivalent to:

$$\sum_{i \in \{1,2\}} T^i \big[\phi\big(m(T^i, H^{nm,ne})\big) - \phi\big(m(T^1 + T^2, H^{nm,ne})\big)\big] \leq 0,$$

which is an impossibility. The function $\phi(\cdot)$ is decreasing for all possible markup values for both the MNL and CES cases according to Lemma C.3. Furthermore, for all i, $m(T^1 + T^2) > m(T^i)$, since Nocke and Schutz (2018, Proposition 6) implies that markups are increasing in type for fixed H. Therefore, the sum above is component-wise strictly positive, which is a contradiction. $\qquad\square$

C.4 Proof of Proposition 3

Proof. Immediate from Propositions 5 - 7. $\qquad\square$

C.5   Proof of Proposition 4

Proof. Let $(\underline{E}, T^F)$ be such that (i) the merger is profit-neutral and (ii) consumer surplus is unchanged due to the merger. From (ii), we know that the aggregator is constant at some level H. From (C.6), we also have

$$T^M = T^1 + T^2 + \underline{E} = \begin{cases} H s^M \exp\left(\frac{1}{1-s^M}\right) & \text{(MNL)} \\ H s^M (\sigma-1)^{1-\sigma} \left(\sigma + \frac{s^M}{1-s^M}\right)^{\sigma-1} & \text{(CES)}. \end{cases}$$

Plugging in for T^1 and T^2 again using (C.6) and solving for $\underline{E}$ yields (16) and (18). From (i), we obtain (17) and (19). We derive these expressions by evaluating the profit functions in (12) for the merged firms before and after the merger, plugging into $\pi^M = \pi^1 + \pi^2$, and substituting in for markups using (A.3) and (A.7), for MNL and CES, respectively (see the proof of Proposition 6, which works out the MNL case in more detail). $\square$

C.6   Proof of Proposition 5

Proof. We first show that, for all choices of T^F, there is a unique efficiency E that makes the merger CS-neutral. Fix T^F and suppose that the merger is CS-neutral. Then $H_{nm,ne} = H_{m,e} = H$. Since types are unchanged across market structures, by (10) and (11) it follows that:

$$s^1_{nm,ne} + s^2_{nm,ne} = s^F_{m,e} + s^M_{m,e}. \tag{C.8}$$

This establishes claim (iii). Clearly $s^1_{nm,ne}$ and $s^2_{nm,ne}$ do not depend upon E. Moreover, by (9) and (10), $s^F_{m,e}$ depends only on T^F and H, not E. Then, by appeal to (10) and (A.3), the only term in (C.8) that depends on E is pinned down by:

$$\frac{T^1 + T^2 + E}{H} = s^M_{m,e} \exp\left(\frac{1}{1 - s^M_{m,e}}\right), \tag{C.9}$$

the left-hand side of which is strictly increasing in E. However, by (C.8), the right-hand side does not depend on E and hence there can be only one such value for E.

We now establish that the CS-neutrality curve is downward-sloping. To this end, suppose consumer surplus is unchanged across the nm, ne and m, e equilibria, and hence that (C.9) obtains. By an identical argument, for the entrant F:

$$\frac{T^F}{H} = s^F_{m,e} \exp\left(\frac{1}{1 - s^F_{m,e}}\right). \tag{C.10}$$

Suppose T^F is increased. This does not change H, as it is pinned down by its value in the nm, ne equilibrium (which does not depend upon T^F) and our hypothesis of CS neutrality. Then by (C.10), an increase in T^F leads to a higher equilibrium share $s^F_{m,e}$. But by (C.8) this implies a corresponding, equivalent decrease in $s^M_{m,e}$ as $s^1_{nm,ne}$ and $s^2_{nm,ne}$ do not depend upon T^F or E. By (C.9), we then conclude the CS neutrality curve is downward sloping.

Finally, claim (ii) follows immediately from the above, and the definitions of these objects. $\square$

C.7 Proof of Proposition 6

Proof. We first establish claim (iii). Suppose that the merger is profit-neutral:

$$\pi^1_{nm,ne} + \pi^2_{nm,ne} = \pi^M_{m,e}.$$

By (12), it follows that:

$$\mu^M_{m,e} + 1 = \mu^1_{nm,ne} + \mu^2_{nm,ne}.$$

By substituting using (A.3) and solving for $s^M_{m,e}$ in terms of $s^1_{nm,ne}$ and $s^2_{nm,ne}$, we obtain:

$$s^M_{m,e} = 1 - \frac{(1 - s^1_{nm,ne})(1 - s^1_{nm,ne})}{1 - s^1_{nm,ne}s^2_{nm,ne}}$$

as desired.

We now show that for all values of T^F, there is a unique efficiency E that makes the merger profit-neutral. Suppose then for some T^F, that there exists some efficiency E is such that the merger is profit-neutral. Then by (10) and (A.3), E satisfies:

$$\frac{T^1 + T^2 + E}{H_{m,e}} = s^M_{m,e} \exp\left(\frac{1}{1 - s^M_{m,e}}\right).$$

However, (iii) implies the right-hand side is constant in E, as it is a function solely of the pre-merger equilibrium quantities $s^1_{nm,ne}$ and $s^2_{nm,ne}$, which do not depend on E. In the Online Appendix (p.110) of Nocke and Schutz (2018), it is shown that for any firm f, T^f/H is increasing in T^f. This implies that if there exists any such E, then it is necessarily unique. To show such an E exists, it suffices to show that the left-hand side (i.e. $T^M/H_{m,e}$) is unbounded above in T^M. Suppose, for sake of contradiction, this is not the case. Then as $T^M/H_{m,e}$ is increasing and bounded above, it converges to some limit $K < \infty$. Since $T^M \to \infty$, this implies $\lim_{T^M \to \infty} H_{m,e} = \infty$ as well. Thus for any $g \in \mathcal{F}_{m,e}$, $g \neq M$, (10) and (A.3) imply that $s^g_{m,e} \to 0$. As g was arbitrary, by (11), $s^M_{m,e} \to 1$ and hence by (10) and (A.3) $T^M/H_{m,e} \to \infty$, a contradiction. Thus $\lim_{T^M \to \infty} T^M/H_{m,e} = \infty$, and in particular, for any such T^F, there exists an E such that the merger is profit-neutral.

We now establish claim (i), that the merger profit-neutrality curve is upward sloping. Suppose that, for T^F, E is such that the merger is profit-neutral. Then, as noted prior, E must satisfy:

$$\frac{T^1 + T^2 + E}{H_{m,e}} = s^M_{m,e} \exp\left(\frac{1}{1 - s^M_{m,e}}\right),$$

where, by (iii), the right-hand side is a constant function in E. Suppose T^F increases. This increases $H_{m,e}$. Since the right-hand side of the above is constant in T^F and $H_{m,e}$, for the equality to hold, the unique solution in E must increase (given the left-hand side is increasing and unbounded in E).

For (ii), the first claim follows immediately from the definitions of $\bar{T}^F$. For the latter claim, suppose that $E = \bar{E}$, and observe that if $T^F = 0$, then the merger is profitable. Conversely, suppose $T^F \to \infty$. Then, as shown above, $T^F/H_{m,e} \to \infty$ as well. By (10), $s^F_{m,e} \to 1$, and hence $s^M_{m,e}$ and $\pi^M_{m,e} \to 0$. Thus we conclude that as $T^F \to \infty$, the merged entrant's

47

profits monotonically decreases to 0. Since pre-merger, the entrant is not in the market, the pre-merger profits of the merging entities are unaffected by T^F, there exists some T^F for which $(T^F, \bar{E})$ makes the merger profit-neutral; as $\pi_{m,e}^M$ is globally decreasing in T^F, this $\bar{T}^F$ is unique. $\square$

C.8 Proof of Proposition 7

Proof. We first establish that, for all choices of $T^F > 0$, there is a unique efficiency E that makes the merger cause the entrant to be profit-neutral. Fix T^F and consider the associated nm, e and m, e equilibria. If the entrant's profits are equal across both equilibria, then by Lemma C.1, $H_{nm,e} = H_{m,e} = H$, and:

$$s_{nm,e}^1 + s_{nm,e}^2 = s_{m,e}^M.$$

In the m, e equilibrium:

$$\frac{T^1 + T^2 + E}{H} = s_{m,e}^M \exp\left(\frac{1}{1 - s_{m,e}^M}\right),$$

the left-hand side of which is strictly increasing in E. However, the right hand side is injective in $s_{m,e}^M$, and $s_{m,e}^M$ is fixed by the nm, e equilibrium and hence its equilibrium is fixed under the hypothesis of entrant profit-neutrality. Thus there can be only one E satisfying the above.[34]

We consider now claim (i), that the entrant profit neutrality curve is downward sloping. By Lemma C.1, we know that $H_{nm,e} = H_{m,e} = H$ and $s_{nm,e}^1 + s_{nm,e}^2 = s_{m,e}^M$. In equilibrium:

$$\frac{T^1 + T^2 + E}{H} = s_{m,e}^M \exp\left(\frac{1}{1 - s_{m,e}^M}\right).$$

By Proposition 6 of Nocke and Schutz (2018), an increase in T^F for fixed E leads to a decrease in $s_{nm,e}^1$ and $s_{nm,e}^2$. But this implies a decrease in $s_{m,e}^M$ as it is the sum of these terms. Thus there must be a commensurate decrease in E.

We now establish claim (ii). Consider the following three market structures: $\mathcal{F}_{nm,ne}$, $\mathcal{F}_{nm,e}$, and $\mathcal{F}_{m,e}$. The entry neutrality line is determined by profit-neutrality across $\mathcal{F}_{nm,e}$ and $\mathcal{F}_{m,e}$; the CS neutrality line is determined by surplus remaining constant across $\mathcal{F}_{nm,ne}$ and $\mathcal{F}_{m,e}$. We first claim that if the two curves intersect for some (T^F, E) then $T^F = 0$. By Lemma C.1, $CS_{nm,e} = CS_{m,e}$; by hypothesis, $CS_{m,e} = CS_{nm,ne}$. Hence in particular, $H_{nm,ne} = H_{nm,e} = H$. Then for each $f \in \mathcal{F}_{nm,e} \setminus \{F\}$, we have:

$$s_{nm,e}^f \exp\left(\frac{1}{1 - s_{nm,e}^f}\right) = \frac{T^f}{H} = s_{nm,ne}^f \exp\left(\frac{1}{1 - s_{nm,ne}^f}\right)$$

and hence $s_{nm,e}^f = s_{nm,ne}^f$. By the adding up constraint:

$$\sum_{f \in \mathcal{F}_{nm,ne}} s^f = \sum_{f \in \mathcal{F}_{nm,e}} s^f$$

---

[34]Here, as H is fixed by the nm, e equilibrium value, the left hand side is unbounded in E.

and thus $s^F = 0$ and hence so too is T^F. Thus consider $T^F \to^+ 0$. If $T^F > 0$, then $CS_{nm,e} > CS_{nm,ne}$, however, $\lim_{T^F \to^+ 0} CS_{nm,e} = CS_{nm,ne}$. Thus as $T^F \to^+ 0$, the associated efficiency tends to $\bar{E}$ by definition.

Suppose now that $T^F > 0$. We will establish that the unique E such that (T^F, E) is entrant profit-neutral must be strictly positive. Suppose, for sake of contradiction, that $E = 0$. Since $T^M > \max\{T^1, T^2\}$, following the merger the markups for the merging firms increase. Given marginal costs remain fixed, the corresponding equilibrium prices increase and hence the effect of the merger on H is an unambiguous decrease. But this implies then $\pi^F_{m,e} > \pi^F_{nm,e} > 0$, a contradiction. By an argument analogous to that appearing in the proof of Proposition 6, an E such that the merger is profit-neutral for F must exist, thus we conclude $E > 0$.

Finally, claim (iii) follows from Proposition C.1, and the immediate observation that, ceteris paribus, entry increases consumer surplus. $\qquad \square$

C.9   Proof of Proposition 8

Proof. The proof mirrors that for Proposition 2, but within a nest. Suppose, for purposes of contradiction, there exists an SPE in which firms 1 and 2 merge, and consumers are unharmed. Thus the merger must increase joint profits:

$$\pi^M(T^1 + T^2, H_g^{m,e}) \geq \pi^1(T^1, H_g^{nm,ne}) + \pi^2(T^2, H_g^{nm,ne}),$$

where $H_g^{nm,ne}$ denotes the nest-level aggregator with no merger and no entry, while $H_g^{m,e}$ is the same object but for a merger with entry. The products in all other nests remain the same, meaning that the resulting overall aggregator is a function of activity from nest g, so we have dropped H in order to save on notation.

By hypothesis, consumers are unharmed, hence we have $H_g^{m,e} \geq H_g^{nm,ne}$. Furthermore, profits are decreasing in H_g according to Nocke and Schutz (2018, Proposition 6), extended to NMNL and NCES in their Appendix (pp. 104-106). Therefore, we have

$$\pi^M(T^1 + T^2, H_g^{nm,ne}) \geq \pi^1(T^1, H_g^{nm,ne}) + \pi^2(T^2, H_g^{nm,ne}). \qquad \text{(C.11)}$$

Multiplying the markup and firm share shows that firm profit is given by

$$\pi^f \equiv \begin{cases} \frac{1-\rho}{\alpha} \mu^f \frac{T^f}{H_g} \exp(-\mu^f) \bar{s}_g & \text{(NMNL)} \\[2mm] \frac{1}{\sigma} \mu^f \frac{T^f}{H_g^{\frac{\gamma-\sigma}{1-\sigma}} H} \left(1 - \frac{\mu^f}{\sigma}\right)^{\sigma-1} & \text{(NCES).} \end{cases}$$

Substituting for profit in the inequality expression C.11 with $\phi(\cdot)$ from (C.1) and canceling gives the condition

$$(T^1 + T^2)\phi\big(m(T^1 + T^2, H_g^{nm,ne})\big) \geq T^1\phi\big(m(T^1, H_g^{nm,ne})\big) + T^2\phi\big(m(T^2, H_g^{nm,ne})\big),$$

where $m(\cdot)$ denotes the markup fitting-in function for the NMNL or NCES, as appropriate. The profit inequality in (C.11) is satisfied if and only if this condition holds. Note that this condition is analogous to that in the non-nested proof for Proposition 2. Markups are also increasing in type, all else equal (again referencing Nocke and Schutz (2018, Proposition 6)).

Thus, we also arrive at a contradiction in the nested case as well. □

C.10 Proof of Proposition 9

Traditionally, the continuity of a fixed point as a function of some set of parameters is established via an appeal to an appropriate form of the implicit function theorem. However, this requires one to consider parameters on the interior of their domain, whereas here we wish to establish continuity precisely on the boundary. Thus we instead employ an approach dating back to Mas-Colell (1974) utilizing a generalization of the implicit function theorem known as the regular value theorem (see Hirsch (2012), Theorem 1.4.1) which remains valid for problems on the boundary.

C.10.1 NMNL Preliminaries

We will prove Proposition 9 by first establishing two intermediate technical results. Define:[35]

$$\Omega_g(H, H_g; \rho) = \frac{1}{H_g} \sum_{f \in \mathcal{F}_g} \sum_{j \in \mathcal{J}^f} \exp\left[\frac{\delta_j - \alpha c_j}{1 - \rho} - \tilde{m}^f\left(\frac{\rho}{H_g} + (1 - \rho) \frac{1}{H_g^\rho H} ; \rho \right) \right].$$

where the the function $\tilde{m}^f(X; \rho)$ defined as the solution in μ^f, for fixed ρ to:

$$\frac{\mu^f - 1}{\mu^f} \frac{1}{T^f \exp(-\mu^f)} = X. \tag{C.12}$$

Let: $\Omega : \mathbb{R}_{++}^{G+1} \times [0, 1) \to \mathbb{R}^{G+1}$ via:

$$\Omega\big((H_g)_{g \in \mathcal{G}}, H; \rho\big) = \begin{bmatrix} \Omega^1(H_1, H; \rho) - 1 \\ \vdots \\ \Omega^G(H_G, H; \rho) - 1 \\ 1 + \sum_{g \in \mathcal{G}} H_g^{1-\rho} - H \end{bmatrix}.$$

The set of equilibria, treating ρ as a free parameter, are precisely the solutions to:

$$\Omega\big((H_g)_{g \in \mathcal{G}}, H; \rho\big) = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{C.13}$$

The differential of Ω, evaluated at a solution to (C.13), is of the form:

$$D\Omega\big((H_g)_{g \in \mathcal{G}}, H; \rho\big) = \left(\begin{array}{ccc|c|c} & \Lambda & & \Theta & * \\ \hline (1-\rho)H_1^{-\rho} & \cdots & (1-\rho)H_G^{-\rho} & -1 & -\sum_{g \in \mathcal{G}} H_g^{1-\rho} \ln H_g \end{array} \right) \tag{C.14}$$

---

[35]See equation (xxxi) in Nocke and Schutz (2018) Appendix (p. 70) for reference.

where Λ is a $G \times G$ diagonal matrix with:

$$\Lambda_{gg} = \frac{1}{H_g} \left(\frac{\rho}{H_g} + \frac{\rho(1-\rho)}{H_g^\rho H} \right) B_g - \frac{1}{H_g},$$

and Θ is the $G \times 1$ matrix with:

$$\Theta_g = \frac{\partial \Omega_g}{\partial H} = \frac{(1-\rho)}{H_g^\rho H^2} B_g,$$

where the expression B_g is given by:

$$B_g = \frac{1}{H_g} \sum_{f \in \mathcal{F}_g} \sum_{j \in \mathcal{J}^f} \exp \left[\frac{\delta_j - \alpha c_j}{1 - \rho} - \tilde{m}^f \left(\frac{\rho}{H_g} + (1-\rho) \frac{1}{H_g^\rho H} ; \rho \right) \right] \tilde{m}^{f'} \left(\frac{\rho}{H_g} + \frac{(1-\rho)}{H_g^\rho H} \right).$$

We now turn to our first technical lemma.

Lemma C.4. *For some $\varepsilon > 0$, the differential $D\Omega$, evaluated at any solution to (C.13) with $\rho \in [0, \varepsilon)$, is of rank $G + 1$.*

Proof. We break down the proof into steps.

1. **Rank at least G:** Firstly, by direct observation, the upper-left $G \times G$ block Λ is diagonal. Moreover, each diagonal element is strictly negative (see Nocke and Schutz (2018) Online Appendix, Lemma XXIII proof). Hence the first G columns of $D\Omega$ are linearly independent, evaluated at any solution to (C.13).

2. **Removal of Nuisance Terms:** Suppose we evaluate $D\Omega$ at the unique solution to (C.13) with $\rho = 0$. Then, in particular:

$$\Lambda_{gg}\big|_{\rho=0} = -\frac{1}{H_g},$$

and

$$\Theta_g\big|_{\rho=0} = \frac{1}{H^2} B_g\big|_{\rho=0}.$$

3. **Contradiction Hypothesis:** Suppose, for sake of contradiction, that the $G + 1$st column of $D\Omega$ evaluated at the unique solution to (C.13) where $\rho = 0$ is a linear combination of the first G columns. Then there exist $(a_g)_{g=1}^G$ such that:

$$(\forall g) \quad \Lambda_{gg}\big|_{\rho=0} a_g = \Theta_g\big|_{\rho=0},$$

and which satisfy:

$$\sum_{g=1}^G a_g = -1. \tag{$*$}$$

Using the results of the preceding step, we can back out these weights:

$$(\forall g) \quad a_g = -\frac{H_g}{H^2} B_g\big|_{\rho=0}.$$

4. **Algebra**: Then, plugging in to $(*)$, we obtain:

$$\sum_{g\in\mathcal{G}}\sum_{f\in\mathcal{F}_g}\sum_{j\in\mathcal{J}^f}\exp\left[\delta_j - \alpha c_j - \tilde{m}^f\left(1/H\right)\right]\tilde{m}^{f'}\left(1/H\right) = H^2.$$

Since we're at an equilibrium (i.e. a solution to (C.13)) we can simplify this using the usual system of equations that hold in an equilibrium. In particular:

$$\sum_{g\in\mathcal{G}}\sum_{f\in\mathcal{F}_g}T^f\exp\left(-\mu^f\right)\tilde{m}^{f'}\left(1/H\right) = H^2.$$

5. **Dealing with $\tilde{m}^{f'}$**: Recall $\tilde{m}^f$ is the implicit solution to (C.12). In particular,

$$\frac{d\tilde{m}^f}{dX} = \frac{T^f\tilde{m}^f\exp\left(-\tilde{m}^f\right)}{1 - XT^f\left[\exp\left(-\tilde{m}^f\right) - \tilde{m}^f\exp\left(-\tilde{m}^f\right)\right]}.$$

For the H under consideration, let us define $\mu^f = \tilde{m}^f(1/H)$. Then this derivative, evaluated at $X = 1/H$, is:

$$\frac{T^f\mu^f\exp\left(-\mu^f\right)}{1 - \frac{1}{H}T^f\left[\exp\left(-\mu^f\right) - \mu^f\exp\left(-\mu^f\right)\right]}.$$

Now, as we are working at an equilibrium, it must be the case that $T^f\mu^f\exp\left(-\mu^f\right) = H(\mu^f - 1)$, hence our expression for the derivative at $1/H$ may be simplified to:

$$\frac{H\mu^f(\mu^f - 1)}{1 + \mu^f(\mu^f - 1)}.$$

6. **Simplifying** Plugging in the result of Step 5 into that of Step 4 and dividing both sides by H yields:

$$\sum_{g\in\mathcal{G}}\sum_{f\in\mathcal{F}_g}T^f\exp\left(-\mu^f\right)\left[\frac{\mu^f(\mu^f - 1)}{1 + H\mu^f(\mu^f - 1)}\right] = H.$$

Note that the square bracketed term lies strictly within $[0,1)$ for all $\mu > 1$. Thus:

$$\sum_{g \in \mathcal{G}} \sum_{f \in \mathcal{F}_g} T^f \exp\left(-\mu^f\right) \left[\frac{\mu^f(\mu^f - 1)}{1 + \mu^f(\mu^f - 1)}\right] < \sum_{g \in \mathcal{G}} \sum_{f \in \mathcal{F}_g} T^f \exp\left(-\mu^f\right)$$

$$= \sum_{g \in \mathcal{G}} \sum_{f \in \mathcal{F}_g} H_g s^{f|g}$$

$$= \sum_{g \in \mathcal{G}} H_g \sum_{f \in \mathcal{F}_g} s^{f|g}$$

$$= \sum_{g \in \mathcal{G}} H_g$$

$$< 1 + \sum_{g \in \mathcal{G}} H_g$$

$$= H.$$

Thus $(*)$ can never hold for any (a_g), and the first $G+1$ columns of $D\Omega$, at the solution to (21) where $\rho = 0$, are linearly independent. By continuity of these terms in ρ, the same must be true for some small enough open set of ρ's containing 0, and the result follows.

$\square$

We now establish the following immediate corollary:

Lemma C.5. *Let* $\hat{\Omega} : \mathbb{R}_{++}^{G+1} \to \mathbb{R}^{G+1}$ *denote the restriction of* Ω *to the (relatively) open set* $\mathbb{R}_{++}^{G+1} \times \{0\}$. *Then* $D\hat{\Omega}$ *is of full rank at the unique solution to (C.13) in this domain.*

Proof. By direct calculation:

$$D\hat{\Omega} = \left(\begin{array}{ccc|c} & \hat{\Lambda} & & \hat{\Theta} \\ \hline 1 & \cdots & 1 & -1 \end{array} \right)$$

where $\hat{\Lambda}_{gg} = -1/H_g$ and $\hat{\Theta}_g = (1/H^2)B_g|_{\rho=0}$, hence an identical argument to the prior lemma yields the result. $\square$

C.10.2 NCES Preliminaries

For NCES we define the function $\tilde{m}^f(X; \sigma)$ as the solution in μ^f, for fixed σ to:

$$\frac{\mu^f - 1}{\mu^f} \frac{1}{T^f\left(1 - \frac{\mu^f}{\sigma}\right)^{\sigma-1}} = X. \tag{C.15}$$

Define:

$$\Omega_g(H, H_g; \sigma) = \frac{1}{H_g} \sum_{f \in \mathbb{F}_g} \sum_{j \in \mathbb{J}^f} \delta_j c_j^{1-\sigma} \left[1 - \frac{1}{\sigma}\tilde{m}^f\left(\frac{\gamma - \sigma}{\sigma}\frac{1}{H_g} + \frac{\gamma - 1}{\sigma}\frac{1}{H_g^{\frac{\sigma-\gamma}{\sigma-1}}H}\right)\right]^{\sigma-1}. \tag{C.16}$$

53

The solutions to (C.16) are equivalent to solving $\Omega_g(H_g, H; \sigma) = 1$. Let: $\Omega : \mathbb{R}_{++}^{G+1} \times [\gamma, \infty) \to \mathbb{R}^{G+1}$ via:

$$\Omega\big((H_g)_{g\in\mathbb{G}}, H; \sigma\big) = \begin{bmatrix} \Omega^1(H_1, H; \sigma) - 1 \\ \vdots \\ \Omega^G(H_G, H; \sigma) - 1 \\ \sum_{g\in\mathbb{G}} H_g^{\frac{\gamma-1}{\sigma-1}} - H \end{bmatrix}.$$

The set of equilibria, treating σ as a free parameter, are precisely the solutions to:

$$\Omega\big((H_g)_{g\in\mathbb{G}}, H; \sigma\big) = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{C.17}$$

The differential of Ω is of the form:

$$D\Omega\big((H_g)_{g\in\mathbb{G}}, H; \sigma\big) = \left(\begin{array}{ccc|c|c} & & & & \\ & \Lambda & & \Theta & * \\ & & & & \\ \hline \frac{\gamma-1}{\sigma-1}H_1^{\frac{\gamma-\sigma}{\sigma-1}} & \cdots & \frac{\gamma-1}{\sigma-1}H_G^{\frac{\gamma-\sigma}{\sigma-1}} & -1 & * \end{array} \right) \tag{C.18}$$

where Λ is a $G \times G$ diagonal matrix with:

$$\Lambda_{gg} = \frac{-1}{H_g} + \frac{1-\sigma}{\sigma}\left[-\frac{\gamma-\sigma}{\sigma}\frac{1}{H_g^2} + \frac{\gamma-1}{\sigma}\frac{\gamma-\sigma}{\sigma-1}\frac{1}{HH_g^{\frac{1-\gamma+2\sigma}{\sigma-1}}} \right] B_g \tag{C.19}$$

at any solution to (C.17), where:

$$B_g = \frac{1}{H_g}\sum_{f\in\mathbb{F}_g}\sum_{j\in\mathbb{J}^f} \delta_j c_j^{1-\sigma}\left[1 - \frac{1}{\sigma}\tilde{m}^f\left(\frac{\gamma-\sigma}{\sigma}\frac{1}{H_g} + \frac{\gamma-1}{\sigma}\frac{1}{H_g^{\frac{\sigma-\gamma}{\sigma-1}}H}\right)\right]^{\sigma-2} \tilde{m}^{f\prime}\left(\frac{\gamma-\sigma}{\sigma}\frac{1}{H_g} + \frac{\gamma-1}{\sigma}\frac{1}{H_g^{\frac{\sigma-\gamma}{\sigma-1}}H}\right) \tag{C.20}$$

and Θ is a $G \times 1$ matrix with:

$$\Theta_g = \frac{\partial\Omega_g}{\partial H} = -\frac{1-\sigma}{\sigma}\frac{\gamma-1}{\sigma}\frac{1}{H_g^{\frac{\sigma-\gamma}{\sigma-1}}H^2}B_g. \tag{C.21}$$

Lemma C.6. *For some $\varepsilon > 0$, the differential $D\Omega$, evaluated at any solution to (C.17) with $\sigma \in [\gamma, \gamma + \varepsilon)$ is of rank $G + 1$.*

Proof. We again break down the proof into steps.

1. **Rank at least G:** Firstly, by direct observation, the upper-left $G \times G$ block Λ is diagonal. Moreover, each diagonal element is strictly negative (see Nocke and Schutz (2018) Online Appendix, Lemma XXIII proof). Hence the first G columns of $D\Omega$ are linearly independent, evaluated at a solution to (C.17).

2. **Removal of Nuisance Terms:** Suppose we evaluate $D\Omega$ at the unique solution to (C.17)

54

with $\sigma = \gamma$. Then (C.19) becomes:

$$\Lambda_{gg}\big|_{\sigma=\gamma} = -\frac{1}{H_g}$$

and (C.21),

$$\Theta_g\big|_{\sigma=\gamma} = \frac{(\gamma-1)^2}{\gamma^2}\frac{1}{H^2}B_g\big|_{\sigma=\gamma}.$$

3. **Contradiction Hypothesis**: Suppose, for sake of contradiction, that the $G+1$st column of $D\Omega$ is a linear combination of the first G columns when evaluated at the unique solution with $\sigma = \gamma$. Since Λ is diagonal, this means that there exist real numbers $\{a_g\}_{g\in\mathbb{G}}$ such that $a_g\Lambda_{gg}\big|_{\sigma=\gamma} = \Theta_g\big|_{\sigma=\gamma}$ (from the first G rows), and $\sum_g a_g = -1$ (the $G+1$st row). From these equations we can solve for a_g:

$$
\begin{aligned}
a_g &= \frac{\Theta_g\big|_{\sigma=\gamma}}{\Lambda_{gg}\big|_{\sigma=\gamma}} \\
&= -\frac{(\gamma-1)^2}{\gamma^2}\frac{H_g}{H^2}B_g\big|_{\sigma=\gamma}.
\end{aligned}
\tag{C.22}
$$

4. **Algebra**: Plugging in (C.22) for the contradiction hypothesis that $\sum_g a_g = -1$, we obtain:

$$\frac{(\gamma-1)^2}{\gamma^2}\sum_{g\in\mathbb{G}}\sum_{f\in\mathbb{F}_g}\sum_{j\in\mathbb{J}^f}\delta_j c_j^{1-\sigma}\left[1-\frac{1}{\gamma}\tilde{m}^f\left(\frac{\gamma-1}{\gamma}\frac{1}{H}\right)\right]^{\sigma-2}\tilde{m}^{f\prime}\left(\frac{\gamma-1}{\gamma}\frac{1}{H}\right) = H^2. \quad (*)$$

5. **Dealing with $\tilde{m}^{f\prime}$**: Consider the $\tilde{m}^{f\prime}$ term now. We know $\tilde{m}^f(X)$ is the solution (in μ^f) to (C.15). Thus, by direct computation:

$$\frac{d\tilde{m}^f}{dX} = \frac{\tilde{m}^f T^f\left(1-\frac{\tilde{m}^f}{\gamma}\right)^{\gamma-1}}{1-XT^f\left[\left(1-\frac{\tilde{m}^f}{\gamma}\right)^{\gamma-1}-\tilde{m}^f\frac{\gamma-1}{\gamma}\left(1-\frac{\tilde{m}^f}{\gamma}\right)^{\gamma-2}\right]}.
\tag{C.23}$$

Considering some fixed solution to (C.17) at $\sigma = \gamma$, define $\mu^f = \tilde{m}^f\left(((\gamma-1)/\gamma)(1/H)\right)$, and let $X = ((\gamma-1)/\gamma)(1/H)$. Then (C.23) becomes:

$$\frac{\mu^f T^f\left(1-\frac{\mu^f}{\gamma}\right)^{\gamma-1}}{1-\frac{\gamma-1}{\gamma}\frac{1}{H}T^f\left[\left(1-\frac{\mu^f}{\gamma}\right)^{\gamma-1}-\mu^f\frac{\gamma-1}{\gamma}\left(1-\frac{\mu^f}{\gamma}\right)^{\gamma-2}\right]}$$

which, given we are at a solution to (C.17), simplifies to:

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{H(\mu^f-1)}{1-(\mu^f-1)\left[\frac{1}{\mu^f}-\frac{\gamma-1}{\gamma}\frac{1}{1-\mu^f/\gamma}\right]}.$$

6. Simplifying: Plugging in to $(*)$ we obtain:

$$\sum_{g \in \mathbb{G}} \sum_{f \in \mathbb{F}_g} \sum_{j \in \mathbb{J}^f} \delta_j c_j^{1-\sigma} \left[1 - \frac{\mu^f}{\gamma}\right]^{\sigma-2} \frac{(\gamma-1)}{\gamma} \frac{(\mu^f - 1)}{1 - (\mu^f - 1)\left[\frac{1}{\mu^f} - \frac{\gamma-1}{\gamma}\frac{1}{1-\mu^f/\gamma}\right]} = H. \quad \text{(C.24)}$$

Simplifying yields:

$$\sum_{g \in \mathbb{G}} \sum_{f \in \mathbb{F}_g} T^f \left[1 - \frac{\mu^f}{\gamma}\right]^{\sigma-1} \underbrace{\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{1}{1-\frac{\mu^f}{\gamma}}\right) \frac{(\mu^f - 1)}{1 - (\mu^f - 1)\left[\frac{1}{\mu^f} - \frac{\gamma-1}{\gamma}\frac{1}{1-\mu^f/\gamma}\right]}}_{\equiv \chi^f} = H.$$

$$\text{(C.25)}$$

By Lemma C.2, in any solution $\mu^f \in [1, \gamma)$ and hence for all g and all $f \in \mathcal{F}_g$, χ^f lives within $[0, 1)$. Thus, considering the left-hand side of (C.25):

$$\sum_{g \in \mathbb{G}} \sum_{f \in \mathbb{F}_g} T^f \left[1 - \frac{\mu^f}{\gamma}\right]^{\sigma-1} \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{1}{1-\frac{\mu^f}{\gamma}}\right) \frac{(\mu^f - 1)}{1 - (\mu^f - 1)\left[\frac{1}{\mu^f} - \frac{\gamma-1}{\gamma}\frac{1}{1-\mu^f/\gamma}\right]}$$

$$< \sum_{g \in \mathbb{G}} \sum_{f \in \mathbb{F}_g} T^f \left[1 - \frac{\mu^f}{\gamma}\right]^{\sigma-1}$$

$$= \sum_{g \in \mathbb{G}} \sum_{f \in \mathbb{F}_g} H_g s^{f|g}$$

$$= \sum_{g \in \mathbb{G}} H_g$$

$$= H,$$

a contradiction of (C.25). Thus the Jacobian $D\Omega$, evaluated at any solution to (C.17) with $\sigma = \gamma$, is of full rank.

$\square$

Lemma C.7. *Let* $\hat{\Omega} : \mathbb{R}_{++}^{G+1} \to \mathbb{R}^{G+1}$ *denote the restriction of* Ω *to the (relatively) open set* $\mathbb{R}_{++}^{G+1} \times \{\gamma\}$. *Then* $D\hat{\Omega}$ *is of full rank at the unique solution to* (C.17) *in this domain.*

Proof. By direct calculation:

$$D\hat{\Omega} = \left(\begin{array}{ccc|c} & \hat{\Lambda} & & \hat{\Theta} \\ \hline 1 & \cdots & 1 & -1 \end{array}\right)$$

where $\hat{\Lambda}_{gg} = -1/H_g$ and $\hat{\Theta}_g = ((\gamma-1)^2/\gamma^2)(1/H^2)B_g\big|_{\sigma=\gamma}$, hence an identical argument to the prior lemma yields the result. $\square$

C.10.3 Proof of Proposition 9

Proof. We state the proof for the NMNL case; the NCES case follows, *mutatis mutandis*. Let $\varepsilon > 0$ be any such value such that the conclusions of Lemmas C.4 and C.5 hold, and by

abuse of notation, denote the restriction of Ω to $\mathbb{R}_{++}^{G+1} \times [0, \varepsilon')$ for any $0 < \varepsilon' < \varepsilon$ simply by Ω. By Lemma C.4, 0 is a regular value of Ω on this domain, and by Lemma C.5, 0 is also a regular value of Ω restricted to the boundary of this domain. Thus by the Regular Value Theorem (see Hirsch (2012) Theorem 1.4.1, see also Mas-Colell (1974) Theorem 2), $\Omega^{-1}(0)$ is a C^1 submanifold of $\mathbb{R}_{++}^{G+1} \times [0, \varepsilon')$, with boundary precisely equal to the unique equilibrium at $\rho = 0$. Consider the (necessarily unique) connected component of $\Omega^{-1}(0)$ that intersects $\mathbb{R}_{++}^{G+1} \times \{0\}$. Since this component is a connected C^1 manifold with boundary, it is C^1-diffeomorphic to $[0, 1)$ (Hirsch (2012) Exercise 1.5.9).[36] Since the Regular Value Theorem guarantees its intersection with the slice $\mathbb{R}_{++}^{G+1} \times \{0\}$ is transverse, the restriction of this component to $\mathbb{R}_{++}^{G+1} \times [0, \varepsilon'']$ for some $0 < \varepsilon'' < \varepsilon'$ is diffeomorphic to $[0, 1]$, and hence is compact.

However, $\Omega^{-1}(0)|_{\mathbb{R}_{++}^{G+1} \times [0, \varepsilon'']}$ is also the graph of the function $e : [0, \varepsilon''] \to \mathbb{R}_{++}^{G+1}$ that takes a nesting parameter value and maps it to the unique equilibrium of the associated differentiated Bertrand-Nash pricing game. By the preceding argument, we may without loss restrict the codomain of e to be some compacta $K \subseteq \mathbb{R}_{++}^{G+1}$ such that (i) $K \times [0, \varepsilon'']$ contains $\Omega^{-1}(0)|_{\mathbb{R}_{++}^{G+1} \times [0, \varepsilon'']}$, and (ii) the graph of e is a closed subset of $K \times [0, \varepsilon'']$.[37] But then by the Closed Graph Theorem (Aliprantis and Border (2006) Theorem 2.58), this map is continuous on $[0, \varepsilon'']$. $\qquad\square$

C.11 Proof of Proposition 10

Proof. We know from Proposition 2 that in the MNL and CES cases, if consumer welfare remains unchanged after a merger, then the profits of the merging firms must fall. Thus, in the NMNL (resp. NCES) model, for $\rho = 0$ (resp. $\sigma = \gamma$), if consumer surplus is unhurt then:

$$\pi^M(T^1 + T^2, H_g^{m,e}, H) < \pi^1(T^1, H_g^{nm,ne}, H) + \pi^2(T^2, H_g^{nm,ne}, H)$$

Suppose then that we consider a sequence of nesting parameter values $(\rho_n)_{n \in \mathbb{N}}$ such that $\rho_n \to 0$ (resp. $(\sigma_n)_{n \in \mathbb{N}}$ such that $\sigma_n \to \gamma$). By Proposition 9, and the continuous dependence of profits and markups on the underlying equilibrium variables $(H_g)_{g \in \mathcal{G}}$ and H, we obtain a sequence of profits for the individual merging parties and the merged entity which converge to their $\rho = 0$ values as $\rho_n \to 0$ (resp. $\sigma = \gamma$ values as $\sigma_n \to \gamma$). In the NMNL model, for n large enough it then must be the case that:

$$\pi^M(T^1 + T^2, H_g^{m,e}(\rho_n), H(\rho_n)) < \pi^1(T^1, H_g^{nm,ne}(\rho_n), H(\rho_n)) + \pi^2(T^2, H_g^{nm,ne}(\rho_n), H(\rho_n)),$$

establishing the result. The sequence of profits would generate an analogous inequality in the NCES model. $\qquad\square$

---

[36]It cannot be diffeomorphic to $[0, 1]$ as from the Regular Value theorem, its boundary is given precisely by its intersection with the boundary of the domain, and at $\rho = 0$ the equilibrium is unique.

[37]It suffices to let K be the projection of $\Omega^{-1}(0)|_{\mathbb{R}_{++}^{G+1} \times [0, \varepsilon'']}$ onto $\mathbb{R}_{++}^{G+1}$ to satisfy both these properties. In particular, this set is compact by continuity of the projection, and the graph of e is closed in $\mathbb{R}_{++}^{G+1} \times [0, \varepsilon'']$ hence it is closed in the subspace topology on $K \times [0, \varepsilon'']$.

C.12 Proof of Proposition B.1

For brevity we focus on MNL demand. An analogous proof for CES demand can be provided upon request by the authors. We first show that as the type of any one firm goes to infinity, so too does the market aggregator.

Lemma C.8. *Fix any market structure $\mathcal{F}_*$ and vector of model primitives. For any $f \in \mathcal{F}_*$,*

$$\lim_{T^f \to \infty} H_* = \infty$$

Proof. First note that $\lim_{T^f \to \infty} T^f / H_* = \infty$, as established in the proof of Proposition 6. Thus, as

$$s^f = \frac{T^f}{H_*} \exp\left[\frac{-1}{1 - s^f}\right],$$

as T^f goes to infinity, s^f goes to one. But as:

$$\frac{1}{H_*} + \sum_{f \in \mathcal{F}_*} s^f = 1,$$

it follows that $H_* \to \infty$. $\qquad \square$

We now prove the proposition.

Proof. Fix an arbitrary market structure $\mathcal{F}_{nm,ne}$ and associated $\mathcal{F}_{m,ne}$. The merger is profitable with delayed or probabilistic entry if and only if:

$$(1 - \theta)\left[\frac{1}{1 - s^M_{m,ne}}\right] + \theta\left[\frac{1}{1 - s^M_{m,e}}\right] \geq \frac{1}{1 - \underline{s}^M_{m,e}} \qquad \text{(C.26)}$$

where

$$\underline{s}^M_{m,e} = 1 - \frac{(1 - s^1)(1 - s^2)}{1 - s^1 s^2}.$$

is the market share of the merged firm in a counterfactual with entry that makes the merger exactly neutral for stage-game profit.[38] We obtain (C.26) by substituting in for profit using (12) and (A.3). Note that $f(x) = \frac{1}{1-x}$ is increasing, and as $s^M_{m,ne} > \underline{s}^M_{m,e}$,

$$\frac{1}{1 - s^M_{m,ne}} > \frac{1}{1 - \underline{s}^M_{m,e}}.$$

Define:

$$(1 - \theta^*) \equiv \frac{\frac{1}{1 - \underline{s}^M_{m,e}}}{\frac{1}{1 - s^M_{m,ne}}}.$$

---

[38] We derive the expression for $\underline{s}^M_{m,e}$ in Proposition 6.

58

Thus for the choice $\theta = \theta^*$, the profit inequality reduces to:

$$\theta^* \left[\frac{1}{1 - s_{m,e}^M} \right] \geq 0,$$

which always holds strictly. The definition of θ^* does not depend on T^F. Thus, for $\theta = \theta^*$, the type of the entrant does not affect whether the merger is profitable. Assuming that $\theta = \theta^*$, we turn to consumer surplus, which weakly increases if and only if:

$$(1 - \theta^*) \ln H_{m,ne} + \theta^* \ln H_{m,e} \geq \ln H_{nm,ne}$$

The only term in this inequality that depends on T^F is $H_{m,e}$. Furthermore, if we send T^F to infinity, then $H_{m,e}$ also goes to infinity, by Lemma C.8. Therefore, for some large enough T^F, and for $\theta = \theta^*$, the merger is profitable and consumer surplus strictly increases. $\qquad\square$

D   Numerical Methods

In this appendix, we describe how a model of Bertrand competition with MNL demand can be calibrated based on data on market shares, and then simulated to obtain the percentage changes in markups, profit, and consumer surplus due to a merger. The NMNL is analogous if one has knowledge of the nesting parameter. We then detail the data sources and methods that are used in the application to the T-Mobile/Sprint merger that is presented in Section 4.

D.1   Calibration and Simulation

With MNL demand, it is possible to recover types from market shares, and vice-versa. To implement the former—a calibration step—first obtain the market aggregator from (11), and the ι-markups from (A.3). Firm types then are given by a rearranged (10):

$$T^f = \frac{s^f H}{\exp\left(-\mu^f\right)}.$$

To implement the latter—a simulation step—use a nonlinear equation solver to recover the shares and the market aggregator, given a set of types. There are $F + 1$ nonlinear equations that must be solved simultaneously. One of these is the adding-up constraint of (11), and the others are obtained by plugging (A.3) into (10), which yields

$$s^f = \frac{T^f}{H} \exp\left(-\frac{1}{1 - s^f}\right).$$

If one knows the types, and thus also the aggregator, then markups, profit, and consumer surplus are identified up to a multiplicative constant (see (9), (12), and (13)). An implication is that the outcomes that arise with different firm types can be meaningfully compared—the ratio of outcomes is identified because the multiplicative constant cancels.

A full calibration also recovers the multiplicative constant—the price parameter, α. This can be accomplished with data on one margin, for example. See also the Nocke and Schutz

(2018) Online Appendix. Then markups, profit, and consumer surplus also are obtained (not just up to a multiplicative constant). However, these objects are not necessary for our purposes, so we use partial calibration.

An observation is that our market shares, $\{s^f\}\ \forall f \in \mathcal{F}$, assign a positive share to the outside good. Thus, they differ from the antitrust market shares described in the US Horizontal Merger Guidelines, which assign zero weight to products that are outside the relevant market.[39] Nonetheless, it is possible to convert antitrust market shares into our market shares using information that often is available during merger review. For example, suppose one has information on the diversion ratio that characterizes substitution from firm k to firm j. Then, in the context of MNL (and CES) we have

$$\frac{\frac{\partial s^j}{\partial p^k}}{\frac{\partial s^k}{\partial p^k}} \equiv DIV_{k \to j} = \frac{s_j}{1 - s_k}. \tag{D.1}$$

Letting the relevant antitrust market comprise the products of firms $f \in \mathcal{F}$, we have

$$\hat{s}^f = \frac{s^f}{1 - s^0} \tag{D.2}$$

where $\hat{s}^f$ is the antitrust market share and s^0 is the outside good share in our context. The system of equations in (D.1) and (D.2) identifies s^0 and $\{s^f\}\ \forall f \in \mathcal{F}$ from data on diversion, $DIV_{k \to j}$, for some $j \neq k$, and the antitrust market shares, $\{\hat{s}^f\}\ \forall f \in \mathcal{F}$.

D.2   Application to T-Mobile/Sprint

Our primary source of data is the 2017 Annual Report of the FCC on competition in the mobile wireless sector.[40] We obtain the following information:

- Among national providers, Verizon, AT&T, T-Mobile, and Sprint account for 35.0, 32.4, 17.1, and 14.3 percent of total connections at end-of-year 2016, respectively. See Figure II.B.1 on page 15.

- The average revenue per user (ARPU) in 2016:Q4 for Verizon, AT&T, T-Mobile, and Sprint is 37.52, 36.58, 33.80, and 32.03, respectively. See Figure III.A.1 on page 42. Following common practice, we use the ARPU as a measure of price.

- The EBITDA per subscriber in 2016 for Verizon, AT&T, T-Mobile, and Sprint is 22.71, 18.30, 11.80, and 13.00, respectively. See Figure II.D.1 on page 24. We interpret the EBITDA as providing the markup.

Finally, we obtain a market elasticity of -0.3 from regulatory filings.[41] The market elasticity is defined theoretically as $\epsilon = -\alpha s_0 \bar{p}$, where $\bar{p}$ is the weighted-average price.

---

[39] See the US Horizontal Merger Guidelines §5 for a discussion of market shares.

[40] *Annual Report and Analysis of Competitive Market Conditions With Respect to Mobile Wireless, Including Commercial Mobile Services*, FCC-17-126.

[41] Specifically, we reference Appendix F of the 2018 Joint Opposition Filing by T-Mobile and Sprint in FCC WT Docket No. 18-197.

The main distinction between the T-Mobile/Sprint application and our other numerical results is that we do not observe pre-merger market shares. The reason is that the FCC data on total connections does not incorporate the consumer option to purchase the outside good. Thus, we use a full calibration approach with the market elasticity and a markup (specifically that of T-Mobile) to recover the outside good share and the price coefficient. We obtain an outside good share of 0.084. With this in hand, the pre-merger market share for T-Mobile, for example, is $17.1/(1 - 0.084)$. With the pre-merger market shares, Figure 7 can be created using the methods described above.